DOCUMENT RESUME

ED 050 072 SP 007 208

TITLE	Mathematics Guide, K-12.
INSTITUTION	Volusia County Board of Public Instruction, De Land, Fla.
NOTE	98p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	*Curriculum Guides, *Elementary Grades, *Kindergarten, *Mathematics Curriculum, *Secondary Grades

ABSTRACT

GRADES OR AGES: K-12. SUBJECT MATTER: Mathematics. ORGANIZATION AND PHYSICAL APPEARANCE: The guide is divided into numerous straight-text chapters interspersed with diagrams and charts. It is xeroxed and spiral-bound with a paper cover. OBJECTIVES AND ACTIVITIES: General objectives for mathematics are outlined in an introductory section. More specific objectives are listed for three levels: grades K-6, 7-9, and 10-12. Subsequent chapters present a method for grouping students into four levels on the basis of ability and for selecting textbooks for each level. Detailed content sequence charts for grades K-6 keyed to two different textbook series are included. Content suggestions for grades 7-9 and 10-12 are brief and general. Several appendixes contain lists of suggestions for mathematics projects. No mention is made of appropriate grade or ability level for these activities. A special section gives hints on helping slow learners. INSTRUCTIONAL MATERIALS: No mention, except of standard textbooks. STUDENT ASSESSMENT: Guidelines suggest the use of both standardized and teacher-made tests. Several sample diagnostic tests are included. (RT)

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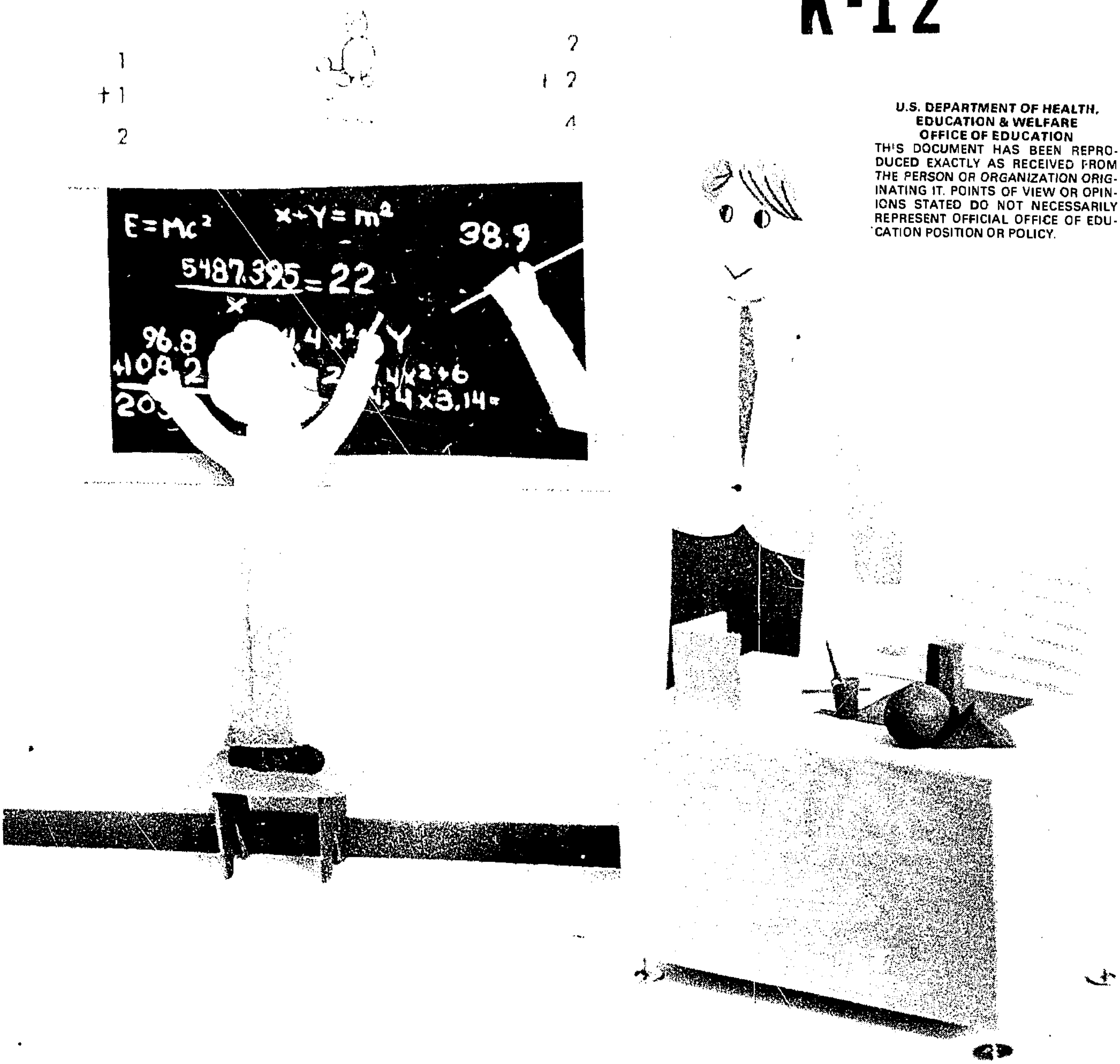
VOLUSIA COUNTY MATHEMATICS GUIDE

K-12



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SUPERINTENDENT OF SCHOOLS
DE LAND, FLORIDA

FOREWORD

The sound development of the instructional program for Volusia County Schools requires a continual re-orientation of teaching methods to maintain contact with the most recent developments in the field. In mathematics many of the recent advancements have been dramatic departures from the traditional pattern and have been warmly welcomed by the informed student and teacher.

This guide to the teaching of mathematics is an attempt to categorize for the moment the status of Mathematics Education in Volusia County. This material will serve to inform parents, interested patrons and new teachers of the nature of our program. As additional changes occur in teaching concepts and materials, these changes will be reflected in revisions and updated versions of this guide.

The contributions of the many individuals who cooperated to make this guide possible are gratefully acknowledged and much appreciated.

Raymond G. Dunne

Raymond G. Dunne
County Superintendent

Little need be said about the growing importance of mathematics in our modern society. Practically every enterprise, large or small, requires its use. For this reason and others, it is important that those of us involved in mathematics education pay special attention to the development and improvement of this part of the curriculum.

The main purpose of this guide is to deal primarily with the many problems related to mathematics education and to offer, where possible, some suggestions for dealing with each problem.

Many experienced teachers and staff members helped develop this guide through a Stetson University sponsored PRIDE course. The guidelines adopted at the outset were simple:

1. To give practical help to teachers.
2. To be clear, concise, and decisive.
3. To deal primarily with methodology, since the state guides, departmental guides, and textbooks cover content.

It must be emphasized that this guide is a beginning, not an end. It should be continually reviewed and improved so that it will reflect the best thinking of contemporary educators regarding mathematics education in particular and the education of children in general.

Francis T. Sganga, Coordinator

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ACKNOWLEDGEMENTS

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VOLUSIA COUNTY, FLORIDA

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EDUCATIONAL PHILOSOPHY

of the

Volusia County School System

Education is the bulwark of a democracy, the best assurance of a progressing society, and the open door to the individual for the realization of his potentialities.

The objectives of education are to develop productive, efficient, and socially minded citizens by providing opportunities for maximum personal development which includes mastering basic skills, knowledges, appreciations, and understandings.

It places proper emphasis upon mental, physical, emotional, spiritual, and social developments of the individual in his relationship to himself and in his relationship to society.

In order to accomplish these objectives, the VOLUSIA COUNTY SCHOOL SYSTEM believes that educational opportunities should be provided to people of all age levels from kindergarten through adulthood.

In conjunction with the above, we further believe that:

The foundation of our society rests upon the value we place upon every individual in it. As the quality of each person is improved, so too does our society improve by the same measure. For this reason, it is incumbent upon all of us who influence the lives of children, to do so in the most constructive manner possible. Consequently, the practical intent of this guide is to help increase the worth of every individual teacher and student that may come within range of its influence. In this way, we hope to make as great a contribution as possible toward the betterment of our society.

MAJOR OBJECTIVES OF MATHEMATICS EDUCATION

The major objectives of MATHEMATICS EDUCATION encompass three phases. These are MATHEMATICAL, SOCIAL, and CULTURAL:

The mathematical phase involves understanding the structure of mathematics and its underlying principles, number systems, and the properties of numbers.

The social phase includes the uses of mathematics in such areas as business, science, and every day living.

The cultural phase involves an appreciation of the historical development of mathematics, its involvement in art and architecture, and its innate universal orderliness.

It will be the objective of this guide to focus each teacher's attention upon these three aspects of mathematical education so as to help each student gain the depth and mastery of the details best suited to his own development and to the needs of society.

Through the suggestions incorporated in this guide, we hope each student will:

1. Develop an understanding and appreciation of our number system and learn to enjoy working with number relationships.
2. Become skilled in the kinds of computations that are basic to the further study of mathematics and its application to science, the business world and daily living.
3. Become aware of the exciting creative side of mathematics.
4. Become a self-directed individual who will continue to learn on his own initiative.
5. Learn to do effective quantitative thinking in coping with problems related to school and everyday living.

WHAT IS THE RELATIVE IMPORTANCE OF MATHEMATICS IN THE LIVES OF STUDENTS?

True learning can only occur through the art of communication, and this art implies that there must be a sender, a receiver, and a "vehicle." In school, the primary sender is the teacher, the receiver is the student, and the vehicle is the spoken word. Yet, even if all three of these conditions are met, does communication necessarily follow? Suppose the sender spoke French to students who only understood English? Evidently, another requirement is vital to the art of communication. There must be UNDERSTANDING.

The ability to recognize, use and understand words is the foundation of all learning. In school, the LANGUAGE ARTS teacher bears the major responsibility for developing these skills. How well each student masters his language arts skills will determine to a large extent how well he does in school. To "fail" in language arts would be tantamount to failing in school.

Second in importance to the language arts skills are the skills related to the area of MATHEMATICS. Today, the need for mathematically literate citizens is greater than ever before. More and more people are buying cars, assuming mortgages and purchasing all types of insurance. Add to this the need for all informed citizens to know something about personal, local, state and federal budgets, social security, medicare, income taxes, real estate taxes, mutual funds, and the interest rates on savings and loans, and we can readily see how important mathematics has become in our daily lives. Yet this is but one aspect of the role of mathematics in our society. The other centers around the growing need for mathematically competent people in a nation that leads the world in its technological advances. In today's labor market, two-thirds of the skilled and semi-skilled job opportunities depend upon an understanding of the basic principles of arithmetic, elementary algebra, and geometry.

THE RESPONSIBILITIES OF THE TEACHER TO THE LEARNER

Those who are well have no need of a physician, but those who are sick.
Matthew 9:12

When a student succeeds in school, who is responsible? When a student FAILS, who is responsible? As any experienced teacher knows, there can be many underlying reasons for success or failure in school. However, it is important to ask these questions of ourselves, particularly when a student fails because failure has such a damaging effect upon individuals.

While we realize we "can't win them all," it is nevertheless incumbent upon us to try our utmost to prevent student failures. One of the first steps we should take is that of self-examination. Is it proper to assume the attitude that it is SOLELY the student's responsibility to learn, and that if he shirks this responsibility, he deserves the consequences? What if the teacher is the only hope for survival in school? Should a knowledgeable and mature adult allow an inexperienced youngster to pursue a course that would be detrimental to both himself and society?

In the final analysis, we must decide for ourselves the course we wish to follow. Experienced teachers have found that many potential failures can be prevented in alarmingly simple ways. Some of the following suggestions may make the difference:

1. Show your students that you have a personal interest in their welfare; that you CARE about them. This may involve no more than an occasional pat on the back with a "How're you doing, John?"

2. Check PERMANENT RECORDS for standardized test scores and past performance to give you a better understanding of each student's potential.

3. Give DIAGNOSTIC TESTS during the first week of school. Go over these with your students. Suggest course changes, if you deem it advisable. (Page 50)

4. Provide enrichment activities for faster students. (Page 79)

5. Provide learning experiences in which there is a chance for SUCCESS. This is most important in mathematics where abilities vary so much. It is useless to try to force the learning of fractions upon a student who cannot multiply whole numbers.

6. Develop a repertoire of MOTIVATIONAL TECHNIQUES. Talk to fellow teachers, read professional literature on the subject and try out several of your own. (Page 68)

7. Continually EVALUATE the progress of students via oral questioning, frequent short quizzes and formal tests. (Page 46)

8. Watch for early signs of trouble. Parents should be informed as soon as possible, and as far in advance of the six weeks grading period as practical.

9. Have students stay after school for help sessions.

10. Confer with parents. Advise them on how they can help their children.

11. Be firm, fair and friendly. Everyone appreciates a word of approval, especially when difficult tasks confront us.

12. Be wary of favoring only the fast students. No relation has been found to exist between grades and success in life. Actually, how many of us were "fast" students in all subjects?

The relationship between students and teachers is a critical factor in the process of education. Studies have shown that students react most favorably to the warm, friendly approach. Personality traits, such as a good sense of humor, ranked higher than specific skill in teaching. The most disliked teacher was the overly-domineering, authoritarian person who failed to respect the personal integrity of his students.

> "Parenthetically, it is interesting to meditate upon Dr. Ernest O. Melby's observation that the teaching profession stands alone in that its members wish to avoid those who need them most. In medicine, the physician deals almost exclusively with the sick; and the more esoteric the illness, the more professional interest he has in it."*

[1] Katharine J. S. Sasse, "Mathematics for the Non-College Bound in Junior High School." THE MATHEMATICS TEACHER, March, 1965, (232-240)

THE ROLE OF THE PRINCIPAL IN MATHEMATICS EDUCATION

The importance of the principal's position is readily apparent when we look at his responsibilities. He is expected to provide leadership for curriculum development, teacher supervision and evaluation, staffing, pupil progress, financial and academic records, discipline, guidance, transportation, public relations, overall school evaluation, budgeting of school funds, departmental meetings, and so on. Yet, as he performs these many tasks, he needs to be constantly aware of the fact that the instructional program must be his primary concern. The old educational adage still holds: "As the principal goes, so goes the school." Therefore, it is important that the principal not only be well informed about what is happening in his school, but that he also keep up with what is going on in general in the field of education.

What role can the principal play in such a specialized area as mathematics? One of his first tasks is the wise selection of teachers who are not only competent in mathematics but are also attuned to the general needs of students. Few students become mathematicians. The principal should make sure the prospective mathematics teacher understands that there is more to teaching than following the book; that teaching concepts and relationships must supercede the teaching of facts. The "tell-drill-test" kind of teacher is outmoded. How does the prospective teacher plan to involve his students in the learning process? How does he plan to give MEANING to the mathematics being studied? The answers to these and similar questions should help the principal determine the possible effectiveness of a beginning teacher.

In addition, the principal can help improve mathematics instruction in the following ways:

1. Be well informed and conversant with regard to current trends in mathematics education. A good source of information should be his mathematics chairman or the mathematics coordinator.

2. Select a competent mathematics chairman who has a broad understanding of the needs of students.

3. Make provisions for meetings to be held at the discretion of departmental or school chairman. Also, provide for an evaluation of the mathematics program with teachers, the supervisor, the guidance department and mathematics coordinator.

4. See to it that each teacher has a copy of the mathematics portion of THE ACCREDITATION STANDARDS FOR FLORIDA SCHOOLS, and that each is well-informed regarding its contents.

5. Have on file the PHILOSOPHY of the mathematics department and the proposed steps for implementing the philosophy.

6. Secure and review copies of formal tests being given to gain some insight into the natur the mathematics being taught.

7. Encourage teachers to use the information in students' permanent records, especially standardized test results. The overall results should be discussed during a regular meeting when the principal and coordinator can be in attendance.

8. Where homogeneous grouping is practiced, classes containing slow students should be kept as small as possible.

9. Encourage the use of concrete objects and good instructional aids at all levels of instruction by recommending that the budget committee allocate a reasonable portion of available funds consistent with the needs of the mathematics department.

10. Encourage teachers to try out new methods and materials. However, he should ask the teacher to keep him informed of what is going on and to submit a brief report of the results. Also, an occasional classroom visit would show the teacher that the principal has a genuine interest in what is being done.

11. While teachers should be encouraged to innovate, the principal should be wary of those who press him to get on the bandwagon when new trends develop. He should seek the advice of his mathematics chairman, his supervisor and the mathematics coordinator. Radical changes in curriculum should always be tested on a small scale and should involve as few students as possible.

12. Have the mathematics chairmen develop ways to foster good public relations by keeping parents informed regarding major changes in the mathematics curriculum. A PTA program devoted to mathematics would be helpful in this regard.

13. Work with the mathematics department in the development of policies related to:

    (a) Homework; (b) Grading; (c) Grouping; (d) Promotions.

14. Encourage teachers to recommend improvements. Nothing is done so well that it cannot be improved.

15. Work toward the articulation of programs between the various levels of education.

16. Encourage teachers to belong to the local, state and national mathematics organizations.

17. Have the librarian subscribe to professional mathematics journals, particularly THE ARITHMETIC TEACHER (Grades K-9) and THE MATHEMATICS TEACHER (Grades 7-12).

18. Make certain that all mathematics teachers study the guide. Constructive suggestions are welcomed, and they should be forwarded to the supervisor or mathematics coordinator.

THE ROLE OF THE SUPERVISOR OF INSTRUCTION

In addition to other things, such as problems relating to staffing, the scheduling of classes, and accreditation, the Supervisor of Instruction is responsible for overseeing the development of all areas of the curriculum. Such responsibility dictates that the supervisor be made aware of proposed changes in the mathematics curriculum so that he can have an opportunity to render any assistance that may be needed.

In view of the supervisor's overall responsibility, it is necessary that the curriculum staff members relate to the supervisor suggestions for curriculum improvement as well as evaluations of existing programs.

The supervisor will work with principals in identifying areas of needed improvement both in program and in school staff. Principals will consult with supervisors and curriculum staff members as changes and new approaches are considered at the school level. The main goal is to foster a team approach so that all participants are given an opportunity to contribute their strengths toward the improvement of this very important area of curriculum.

THE ROLE OF THE COUNTY CURRICULUM STAFF MEMBER

1. Work with the principal and supervisor to coordinate K through 12 mathematics offerings, as well as the textbooks used in these offerings.
2. Help coordinate programs between feeder and recipient schools.
3. Work with the principal and mathematics chairman to evaluate programs, and make suggestions for improvements.
4. Meet with faculties and departments to discuss problems and develop possible solutions.
5. To do in-service work with beginning teachers, and plan in-service workshops for all teache
6. Do demonstration teaching; show how good instructional aids can be used effectively.

7. Inform teachers about promising innovative programs, the results of currect research and special projects going on in the county and elsewhere.

8. Coordinate and help in the continual development of VOLUSIA COUNTY'S GUIDE TO MATHEMATICS EDUCATION.

9. Foster professional interest in local, state, and national mathematics organizations by working closely with the officers of the VOLUSIA COUNTY COUNCIL OF TEACHERS OF MATHEMATICS.

10. Help teachers locate and develop materials to enable them to help slow and fast learners as well as the average student.

THE OBJECTIVES OF MATHEMATICS IN THE THREE MAJOR LEVELS OF EDUCATION

There are two interrelated aspects to education: general education and specialized education. How do we differentiate between the mathematical needs of students who fall into these two general categories? WHO is to decide what these needs are, the mathematics specialists or those concerned with the general education of children? The central question is not "Can it be learned?" but rather, "Should everyone have to know it?"

Too often, mathematics is viewed by many students as a self-contained discipline with little relationship to real life. This is particularly true at the secondary level where such students may be able to "solve" problems like the following, but can see little reason for doing it except perhaps to pass a test:

7th Grade	8th Grade	Algebra
$324_8 \times 216_8 = \underline{\qquad}_8$	$\begin{array}{rrrrr} 2b^4 & + & -3b^3 & + & 5 \\ b^2 & + & 2b & + & -3 \\ \hline \end{array}$ (Multiplication)	$\dfrac{1 + \dfrac{x_2}{y - x}}{y - \dfrac{x}{1 - \dfrac{y}{y + x}}}$

The teacher's effectiveness is determined by how well his students learn, and since learning involves the discovery of personal MEANING in whatever is taught, the mere accumulation of facts is not necessarily a learning situation. How many of us have attended institutes and workshops which had little MEANING for us because they were subject-matter oriented and not geared to be of direct help to us in the classroom? What then should our objectives be in mathematics education in a country which is committed to educating all of its people?

OBJECTIVES: K THROUGH 6

Look at the 30 youngsters in your elementary class and consider what the future holds for them. Some may become professional people, such as teachers, lawyers, doctors, engineers and so on. Others may go into any of the thousands of jobs requiring varying degrees of ability, from businessmen, technicians, secretaries to jobs requiring little or no skill at all.

At the K through 6 level of education, it is important to recognize the fact that there are as many personalities and levels of abilities as there are children. It is equally important to realize that each child's vocation will probably be different, and that only a relatively small number will need to be highly skilled in mathematics. But there is the rub. We are faced with the difficult problem of trying to meet the needs of ALL students, and it is our job to try to do this as effectively as possible.

In view of these observations, what should be our objectives at this level? One of the first is to do one's utmost to develop in children a POSITIVE ATTITUDE TOWARD MATHEMATICS. Concomitant with this is the need to insure that each student, regardless of ability, have an opportunity to EXPERIENCE SUCCESS in this vital area of instruction. In addition to these attitudinal objectives are those related to subject matter.

The minimum achievement of the majority of students by the end of the 6th year should include:

1. An understanding of the decimal system of numeration, including the ability to read and write numerals. There should also be an awareness of the fact that there are other systems of numeration, and that all of them have common characteristics.
2. The ability to skillfully perform the four basic operations in whole numbers, fractions, decimals and percents. This would include understanding the interrelatedness of the operations and the way the numbers are expressed.
3. A working knowledge of how to find the solutions to practical problems.
4. An awareness of the cultural, esthetic, and historical aspects of mathematics, especially with regard to the overall contribution mathematics has made to our present standard of living, and the vital role it is continuing to play in our daily lives.

Experienced teachers fully realize that many students cannot achieve the levels of performance stated in the objectives by the end of the sixth grade. These students will need special attention, and ways to provide this attention will be discussed in the section entitled: SOME WAYS TO MEET THE INDIVIDUAL NEEDS OF STUDENTS.

OBJECTIVES: 7 THROUGH 9

Children enter the seventh grade with a wide range of individual differences in mathematical ability. One of the major objectives at this level of development is to carry each youngster forward in mathematical competence from where he is. Hence, if a youngster has yet to master the basic multiplication facts, then this is where he ought to start. But if a youngster has already achieved a high degree of competence in all of the basic operations and is ready for advanced work, then provisions must be made to allow him to work at HIS level of achievement.

Fundamentally, in view of the natural differences to be found among youngsters, it is necessary that we develop methods and materials to accommodate these differences. This will require that we individualize instruction to whatever degree we can so that each student will be able to cope with the tasks assigned him. The homogeneous grouping of classes is one attempt to meet this challenge. However this merely NARROWS the range of differences. Unless such grouping is accompanied by changes in methods and materials, little can be expected from this approach.

Here again at this level there is a need for students to experience SUCCESS in order that they maintain a positive attitude toward mathematics. It would be a mistake to get too rigorous or too abstract at the seventh or even the eighth grade level. There is still a need to use concrete objects, to engage the student in activities and experiments which he can do himself, and to make a special effort to show how INTERESTING mathematics can be. This should be the time for exploration. Surely, the 4 or 5 more years of mathematics available to the student should be ample time to get as rigorous and abstract as one wishes.

During the seventh year, it will be possible to identify the various degrees of mathematical talent and to help each student chart his future course through the remaining grades.

OBJECTIVES: 10 THROUGH 12

The objectives at the senior high school level should be divided into two major categories. These categories are reflected in the 1963 ACCREDITATION STANDARDS FOR FLORIDA SCHOOLS which states that for Level 1:

> "The mathematics program should provide instruction for students planning to attend college and for students planning to terminate their formal education with high school."

Ordinarily, the choices open to students match the two criteria in the above quote. One is the rigorous, mathematical route; the other, the general math route. Currently, through the adoption of a textbook, an effort has been made to develop a liberal arts approach to mathematics. It is possible that this approach will develop into a third choice for those able, college-bound students who would prefer courses which are less rigorous than those now offered.

Essentially then, the objectives for the college-bound student should be:

1. To be mathematically competent enough to be successful in his chosen field in college.
2. To have a positive attitude toward the subject.
3. To have an understanding of the contribution it has made to our civilization.
4. To create within the student a desire for independent study.

The objectives for the non-college bound student should be:

1. To develop his computational skills, and to show him how to use these skills to solve many different kinds of practical problems.

2. To help him understand that skill in mathematics would be a valuable asset when seeking employment, and that the level of employment may well depend upon one's degree of skill in mathematics.

3. To show the use of mathematics in managing a home by working on problems related to mortgages, installment buying, budget keeping, banking, taxation, insurance, and so on.

4. To encourage these students to continue their mathematical studies since many of them cannot be sure that they will not further their education once they graduate from high school.

HOW THE K-6 OBJECTIVES MAY BE REALIZED

At an FCTM meeting in October, 1966, Dr. Irving Dodes, author of MATHEMATICS, A LIBERAL ARTS APPROACH said that the teacher's challenge is to take the math which students need - and make it interesting - rather than take the math they find interesting, and make them think they need it.

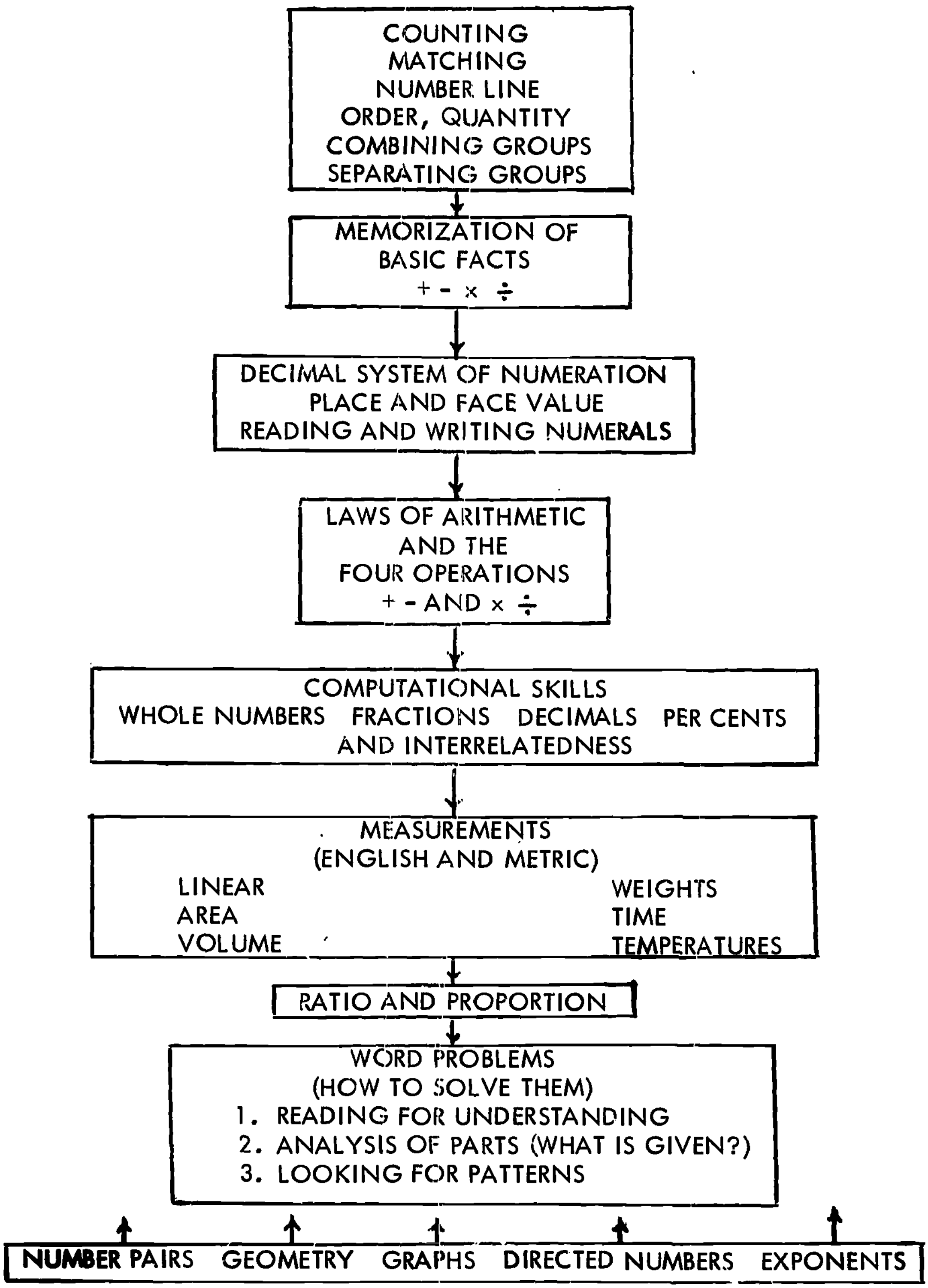
Before we can discuss how K-6 objectives can be realized, it will be necessary to consider the question: "What kind of math is needed by all students?" Obviously, there is little chance that any one answer would suit all of us. However, there ARE major and minor themes in mathematics. Most of us would agree that, comparatively speaking, the concepts of RATIO AND PROPORTION deserve greater attention than operations in number systems other than base ten.

It is with this idea in mind that we have developed the chart on the following page. It is designed to give the teacher an overview of the major areas of importance in elementary mathematics. In most cases it is futile to try to "cover" all of the material in the textbook. For one thing, not all of the material is of equal importance. Of PRIME IMPORTANCE is the memorization of the basic facts as early as possible. The pressures resulting from trying to cover the book could be most frustrating for both students and teacher.

In studying the chart, notice that the arrowheads indicate the most important areas to be stressed. The order of appearance from top to bottom does not imply that topics such as measurements must await treatment of previous topics. The same holds true for WORD PROBLEMS, which is the main reason for studying all of the other areas on the chart.

Notice the five topics at the bottom of the chart. Each of these can be used to "make interesting what the student needs." Some of the material in the appendix of this guide demonstrate how this can be done.

CHART OF MAJOR THEMES FOR ELEMENTARY MATHEMATICS



USING COMMON SENSE TO HELP US AVOID PITFALLS

The school year for students amounts to 180 days. How many teaching days are left after we eliminate the first and last days of school, days for reviewing, testing, field trips, assembly programs, county testing, and so on? After estimating the number of days left, compare your result with the number of pages in your textbook to see how many pages per day would have to be "covered" to finish the book.

We will not consider the pros and cons of "covering the book", since it is generally agreed that this is not the best approach to teaching mathematics, or any other subject for that matter. However, we do want to consider the matter of "time", and how the teacher might use it wisely in relation to the important concepts to be taught during the school year.

One of the most time-consuming tasks is the teaching of algorithms. What is the best way to divide whole numbers? Most current texts show the following ways 58 may be divided by 12:

(A)
$$\begin{array}{rl} 58 & \\ \underline{-12} & (1) \\ 46 & \\ \underline{-12} & (1) \\ 34 & \\ \underline{-12} & (1) \\ 22 & \\ \underline{-12} & (1) \\ 10 & 4 \text{ r } 10 \end{array}$$

(B)
$$\begin{array}{r|l} 12\overline{)58} & 2 \\ \underline{24} & \\ 34 & 2 \\ \underline{24} & \\ 10 & \overline{4} \end{array}$$

(C)
$$\begin{array}{r|l} 12\overline{)58} & 3 \\ \underline{36} & \\ 22 & 1 \\ \underline{12} & \\ 10 & \overline{4} \end{array}$$

(D)
$$\begin{array}{r} 12\overline{)58} \\ \underline{48} \\ 10 \end{array}$$

Which way is best? Which do you require of your students? While (A), (B), and (C) are interesting and are designed to provide students with greater understanding of the process, it is conceded, even in our modern texts, that students must eventually learn all of the short-cuts as shown in (D). Why? For one thing, students must eventually learn all of the short-cuts possible for the practical reason that they will be subjected to standardized tests which place a premium on SPEED. The more items done correctly, the greater the chance for entering the college of your choice, gaining the scholarship you need, or passing the test for getting into any of the military academies.

Another reason centers around the matter of time. There are a certain number of important topics to deal with in a limited amount of time. Because of this, the teacher must ask himself, "How important is the algorithm in relation to the other things I must deal with?" Is it as important as ratios? proportions? problem solving? What is the PURPOSE of an algorithm?

The algorithm is not an end in itself, but a MEANS to an end. Its main purpose is to help in the solution of a problem. Hence, it is primarily a MECHANICAL PROCEDURE fo. achieving an "answer". As an example, let's consider the following problem:

> Gold can be hammered into sheets 0.000005 of an inch thick. How many sheets would it take to make a stack 1/2 inch high?

Complete understanding of the algorithm involved is not critical to the solution of this problem. What IS critical is identifying the algorithm to start with. Does the student MULTIPLY or DIVIDE? If the division algorithm is decided upon, another critical question must be answered: What is divided by what? (This will be dealt with later in HOW TO HELP STUDENTS SOLVE CERTAIN DIFFICULT WORD PROBLEMS.)

Of course, there are many other algorithms, and the same reasoning applies to all of them. They are CALCULATIONS which help us solve problems, and are not solutions in themselves. The "solution" is the correct "setting up" of the problem in the first place. The computation should be practically automatic. The main thing is that it be correct. However, this does not mean that the "whys" of algorithms are to be ignored. Time should be spent in developing an understanding of the "whys" of algorithms. It is the AMOUNT OF TIME ALLOTTED that is in question. NON STANDARD ALGORITHMS SHOULD BE USED FOR **ENRICHMENT ONLY.**

ALLOTTING THE PROPER AMOUNT OF TIME FOR ALL THAT NEEDS TO BE DONE

Each teacher will have to examine his text with respect to the major concepts that are included (see chart on page 16), and try to spread these concepts over a 36 week period. Word problems, computational skill, measurements in all three dimensions, equivalent fractions, ratios, and proportions must take precedence over such areas as number theory, (primes, composites, factorization), computing in bases other than ten, and modular or clock arithmetic. These should be considered as enrichment-type activities or as special areas of study for faster students. It would be helpful to develop a master chart depicting all 36 weeks with the areas of major concern filled in.

TEACHING FOR UNDERSTANDING

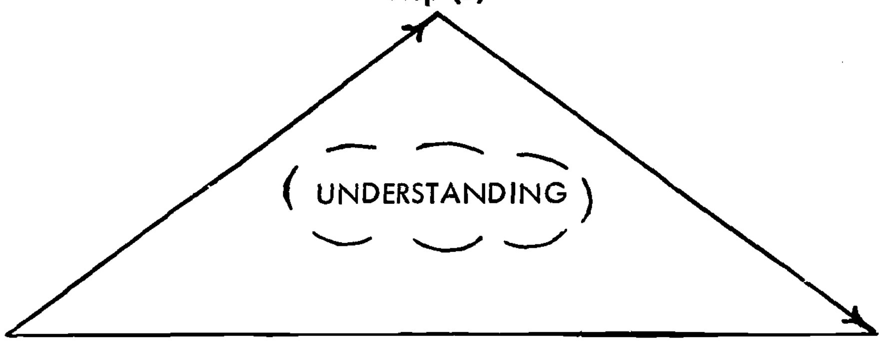
For some reason the impression exists that teaching for "understanding" is a brand new modern concept that has just emerged to slay the dragon, rote memorization. While it may or may not be true that all learning previous to "new math" was learned by rote, an examination of current materials leads us to ask, "Understanding of WHAT?"

In many instances the text material supplies the answer: "Understanding of how ALGORITHMS work." Is this where the greatest amount of effort is to be made? If we assume that an algorithm is basically a MEANS to an end, then we need to expand our thinking regarding "understanding" to include the understanding of HOW TO SOLVE A WIDE VARIETY OF PRACTICAL PROBLEMS. The rise of modern science and the advances of our technological society compel us to pay greater attention to the USEFULNESS of mathematics in dealing with every day problems. We cannot afford to slight this all-important aspect of mathematics education. How then do we help students gain the kind of understanding they need? The chart below may help clarify our thinking:

PRACTICE - DRILL

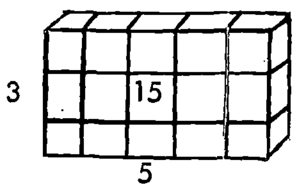
If	$A \times B = C$	Since	$3 \times 5 = 15$
then	$C \div B = A$	then	$15 \div 5 = 3$
and	$C \div A = B$	and	$15 \div 3 = 5$

ABSTRACT
Step (2)



Step (1)
CONCRETE
(Instructional Aids)

CUBIC INCH BLOCKS



Step (3)
APPLICATION
A man owned 15 acres of land and **wished** to sell them in 3 equal **portions.** How many acres would there **be in** each portion?

Youngsters at all levels of instruction can profit from seeing abstract ideas represented by concrete objects. The lower the grade level, the greater the need. With imagination, ordinary cubic inch blocks such as those sketched above can be used effectively at various grade levels. Such questions as: What is a square? A cube? How many "faces" does a cube have? How many edges? What does 2^2 mean? 2^3? may be posed to stimulate thinking in an important area. Having students handle the cubes for themselves would make the exercise that much more effective.

I hear and I forget;
I see **and I remember**;
I do **and I understand.**
Ancient Proverb

DRILL, PRACTICE AND MEMORIZATION

Teaching students to compute skillfully is still one of the major goals of elementary mathematics education. After the teacher has engaged the class in the preliminary developmental work required for understanding, it is then necessary to provide students with the necessary drill (or "practice", if you prefer) for mastery of the process. One of the legitimate complaints about the approach of previous years was that students were assigned too many examples of the same kind. In many instances, more imaginative approaches are used in the new texts. For example, where it was customary to memorize the fact that "6 time 4 is 24", modern texts now challenge the student in the following ways:

1. List the pairs of facts that give the product 24.
2. Place the proper numeral in the box to make this sentence true: $8 \times \square = 24$.
3. Find the number represented by N in: $2 \times N \times 6 = 24$.
4. Solve for the letters shown: (a) $\frac{24}{A} = 3$ (b) $4 = \frac{B}{6}$ (c) $\frac{8}{1} = \frac{C}{3}$
5. Do the part within the parentheses first, then find R: $(4 \times 6) \div R = 8$.

However, once the student understands the concept, it is absolutely necessary for him to commit the basic facts to memory and to be ORALLY quizzed to determine his proficiency. This frees the student from being concerned with routine work so that he can concentrate upon the more interesting aspects of mathematics. He should understand at the outset that to be adept in ANY activity, whether it be football, playing the piano, dancing, or mathematics, **requires that one master the fundamentals** as soon as possible.

SOME WAYS TO MEET THE INDIVIDUAL NEEDS OF STUDENTS: K-6

Many students have shown what teachers have known since group teaching began: there are as many differences among children at any grade level as there are children, and that the spread between the slowest and fastest student increases with grade level advancement. To the experienced and dedicated teacher, this is both a fact and a challenge. How the challenge is being met and what new ways are being devised to meet this challenge will be dealt with in this section. However, it must be understood that an in-depth study of any of the sub-topics would not be possible, since each could well encompass a volume in its own right.

INTERCLASS GROUPING

There are three main factors to be considered in developing any program that is designed to make provisions for individual differences. The first is the desire of the professional staff to do something about meeting the needs of individual students. The second is the need to make a TEAM EFFORT involving teachers, principals, supervisors and math specialists, and the third is to secure or prepare special text materials and teaching aids that are geared to the particular needs of the students.

Interclass grouping, which is designed to "narrow the range" is neither new nor necessarily effective. Quite often, after classes have been homogeneously grouped, all of the students at the same grade level would be handed the same grade level text. A common procedure is to "water down" the text material or teach it at a slower pace in "slow" classes, and go at a "normal" or faster rate in other classes. Is this a wise approach?

The readability of a sixth grade text would probably be a major obstacle to slower students. In addition to this, the grade-level text would contain very little mathematics that would be geared to the achievement levels of the slower students or sufficient practice work for their needs. In view of these facts, a better alternative is to secure sufficient teaching materials, other than the textbook, that are geared to the level of achievement of the students. If workbooks or worktexts are used, they should be non-graded. Many teachers prefer using self-prepared or commercial, single-sheet exercises.

In placing slow learners in a single class, every effort should be made to keep such classes as small as possible. Class size should not exceed 25. Also, special consideration should be given in providing the teacher with many manipulative and demonstration instructional aids, as well as an overhead projector that can remain in the room.

A well-equipped room can be of considerable help to teachers of slow learners. Most important, however, is the need to work as a team to develop the best approach toward helping these children. The resources of the entire faculty through group discussions should be called upon to consider such things as GOALS, MOTIVATIONAL AND INSTRUCTIONAL TECHNIQUES, and DISCIPLINE MEASURES.

In considering goals, it will be necessary to project one's thinking beyond the classroom, and well into the future. A chart, such as the following, may help in this regard:

GOAL: GOOD CITIZENSHIP

↑

A WELL-INFORMED AND CAPABLE PERSON

↑

OTHER ACADEMIC AREAS

↑

LANGUAGE ARTS

The road ahead for slow students and their teachers is a most difficult one. However, if they receive the kind of help they need, and can experience success in what they do, the chances are the goals will be achieved.

GROUPING WITHIN THE CLASSROOM

Much of the same thinking applies to intraclass grouping that applied to homogeneously grouped classes. While grouping for READING instruction is common on the elementary level, it is done in relatively few classes in mathematics. Essentially, it requires the kind of approach used in the one-room school. Each group would be taught using materials suitable for their achievement levels. Separate assignments and progress tests would have to be given, and each student would be allowed to move from one group to another depending upon his performance. In attempting this approach for the first time, the teacher should consult with teachers who have had some experience in setting up such classes, as well as with the principal and county-level personnel. However, an example of some general procedures follow:

1. Present a new concept to the entire class, then divide the class into small groups for drill. Teacher would move between groups as needed.
2. If a fast group finishes quickly and demonstrates a mastery of the concept, its students can be moved into the next lesson on their own, be given enrichment work, or be called upon to help explain the concept to those in the other groups who have yet to understand it.
3. When the slower groups fall too far behind to profit from the work being done by the others, they should be given special text materials to help them at their level of achievement.

TEAM APPROACH

The greatest threat to a teacher's self-confidence is television. Students watch space flights accompanied by technical explanations, they see news reports of world conflicts, and assimilate a wide variety of other information, all of which tends to rank them among the best informed children on earth. "What keeps a satellite up? Why are we fighting on foreign soil? How does a computer work? are questions easily posed but difficult to answer.

In addition to the above, the teacher is expected to competently teach the language arts and all that it encompasses, science via the experimental approach, social studies, perhaps art, music, Spanish, and mathematics. Notice that it is MATHEMATICS and not arithmetic. A close examination of a modern sixth grade text will forcefully give reason enough for the distinction.

The difficulty of teaching modern mathematics is attested to by the fact that teachers by the thousands have attended mathematical institutes to help bring them "up to date", and the fact that there is a movement afoot to require much more college-level mathematics for teachers due to the growing importance of the subject in our society. In a pamphlet called the "Urban Reporter Supplement" (Fall, 1966) published by the NEA, Charles Schulz says:

> "No individual has the competence, energy, and time to deal effectively with all the responsibilities and all the chores traditionally assigned to the teacher. No teacher can operate at maximum effectiveness - as a teacher - in the isolated fashion which has largely characterized the self-contained classroom."

He predicts that:

> "Staff collaboration, at all levels, and school operations geared to team effort will replace traditional patterns.
>
> The one-adult per self-contained classroom pattern will pass out of existence."

Logic and the facts of school life point toward the need to remove some of the pressures from elementary teachers, particularly in the upper grades. One avenue of relief may be found in using a team approach where the interested parties are willing, and conditions permit. In some cases teachers may, for example, exchange responsibilities - one person handling another's science and mathematic programs in exchange for another's language arts and social studies. Some things that need be considered:

1. The desire of the teachers to teach as a team.

2. The professional background and teaching experience of each teacher.

3. Their subject-matter preferences.

4. The willingness to develop, seek out, and use the special materials neede for conducting an effective program. For example, in mathematics, the teacher would accumulate a host of non-graded text materials, good instructional aids, and enrichment materials.

Other team approaches are possible. Once the decision has been made to try to improve mathematics instruction via a team approach, it will be an easy matter to find willing hands at the administrative or staff level to help you work out the details.

SOME WAYS TO MEET THE INDIVIDUAL NEEDS OF STUDENTS: 7-9

The higher the grade level the less the need for grouping within classes. On the junior high school level, homogeneous grouping generally prevails. In the larger schools, there are four major patterns that are designed to help insure that each student completes his course successfully:

Pattern I	HIGH ABILITY
Pattern II	ABOVE AVERAGE
Pattern III	AVERAGE
Pattern IV	BELOW AVERAGE

Note: The patterns are not rigid. Students may move from one to another depending upon his performance.

The determining factors for participation in each pattern are:

1. Achievement scores
2. Intelligence scores
3. Work Record (grades)
4. Maturity
5. Teacher Recommendation

In order that the proper placement of pupils in their mathematics experience sequence may be accomplished, the following criteria have been established for grade seven pupils in their initial year in the junior high school program.

CRITERIA FOR PLACEMENT IN GRADE 7

	ACHIEVEMENT*	I. Q.		GRADE	MATURITY	TEACHER RECOMMENDATION**
		L	N.L.			
Pattern I	8.5 - up	110 up	120 up	A	1	1
Pattern II	7.5 - 8.4	105-109	111-119	A-B	1-2	1-2
Pattern III	5.1 - 7.4	90-104	90-110	B-C	2-4	2-4
Pattern IV	5.0 down	89 down	89 down	D	4-5	4-5

* STANFORD ACHIEVEMENT TEST (Grade 6): Computations, Concepts and Applications
** From 1 (high recommendation) to 5 (low recommendation)

The state department has adopted a sufficient number of textbooks to enable schools to follow more than one approach at any grade level.

The chart which follows matches the books most generally used in our county with the four patterns developed for grades 7-9. It must be kept in mind that where applicable, the texts used in grade 10 should be of the same series as those used in grade 9. Also note where the text REFRESHER ARITHMETIC is located in the chart. This text should not be used at more than one grade level within the same school.

PATTERNS AND SEQUENCE OF COURSES: GRADES 7-9

7th	8th	9th
	PATTERN I: ADVANCED HIGHLY ABSTRACT SEQUENCE	
Mathematics 7 and 8	Algebra I	Geometry
Current Texts: Expl. M.M. Bks. I & II (Keedy)	Mod. Alg. I (Dolciani)	1. Mod. Geom. (Jurgenson) 2. Geometry (Moise)
	PATTERN II: ABOVE AVERAGE MODERATELY ABSTRACT SEQUENCE	
MATH 7	MATH 8	ALGEBRA I
Current Texts: Expl. M.M. Bk. I (Keedy)	Expl. M.M. Bk. II (Keedy)	Mod. Alg. I (Dolciani)
	PATTERN III: AVERAGE TO BELOW AVERAGE MODERATELY ABSTRACT SEQUENCE	
MATH 7	MATH 8	ALGEBRA I or PRE-ALGEBRA**
Current Texts: Mathematics 7 (McSwain)	Mathematics 8 (McSwain)	Modern Alg. (Dolciani) Contemp. Alg. I (Smith) Pre-Alg. Math (Nichols) General Math Bk. I (Brown)
	PATTERN IV: BASIC MINIMALLY ABSTRACT SEQUENCE	
MATH 7	MATH 8	MATH 9
Current Texts:* Mod. Basic Math (Eicholz)	Mod. Basic Math (Eicholz)	Gen. Math Bk. I (Brown) Refresher Arith. (Stein)

* or SPECIAL MATERIALS

** NOT A SUBSTITUTE FOR ALGEBRA

SOME WAYS TO MEET INDIVIDUAL DIFFERENCES: GRADES 10-12

Due to the fact that students may choose several courses of study at the senior high school level grouping tends to occur automatically. Here again, those who have had difficulty with mathematics in the past, are the ones most difficult to reach.

In examining the chart on the following page, keep in mind that it is meant to provide the most general kind of guidelines. Each school will have to develop its own detailed approach to caring for individual differences. One way to do this is through the use of programmed materials.

SEQUENCE OF COURSES

9th	10th	11th	12th
I GEOMETRY Current Texts: 1. Mod. Geom. (Jurgenson) 2. Geometry (Moise)	ALGEBRA II Mod. Alg. II (Dolc.)	TRIG. & ANAL. GEOM Sem. I Mod. Trig. (Welchons) Sem. II Analyt. Geom. (Fuller)	ADVANCED MATH New texts presently up for adoption.
II ALGEBRA I Current Texts: Mod. Alg. I (Dolc.)	GEOMETRY 1. Mod. Geom. (Jurgenson) 2. Geometry	ALGEBRA II 1. Mod. Alg. II 2. Cont. Alg. II	TRIG. & ANAL. GEOM. Sem. I Mod. Trig. (Welchons) Sem. II Anal. Geom. (Fuller)
III ALGEBRA Current Texts: 1. Cont. Alg. I (Smith) 2. Pre-Alg. Math (Nichols)	GEOMETRY 1. Mod. Geom. (Jurgenson) 2. Geometry (Moise)	ALGEBRA II Cont. Alg. II	TRIG. & ANAL. GEOM. Sem. I Mod. Trig. (Welchons) Sem. II Anal. Geom. (Fuller)
IV MATH 9 Current Texts: Gen. Math Bk. I (Brown) Ref. Arith. (Stein)	GENERAL MATH BUSINESS MATH ALGEBRA I		

GENERAL MATHEMATICAL CONTENT: K-6

KINDERGARTEN

No one textbook or worktext should govern the content of any course, nor should the teacher feel bound to any one approach. The important thing is to help children understand the meanings of basic quantitative words (such as big, little, many, few, more, less, etc.), to be involved in quantitative situations (play store and post office) and learn to recognize certain symbols (such as the numerals used for digits). Much of the learning should be informal, and as personal as possible, using and building upon knowledge they already have or should have. Examples of some quantitative questions appropriate for kindergarten children are:

How old are you? What is your birthday?

What is your address? Do you know your phone number?

How many brothers and sisters do you have?

How many hands do you have? How many fingers?

Can you name the numerals on a clock?

Who wants to be first? second? third?

Can you fold the paper in half?

How much do you weigh?

What channel is Bat Man on?

Which page did we finish yesterday?

How many pennies do you have?

Which would you rather have, a penny or a nickel? Why?

How many plan to buy lunch?

How many are absent from the FIRST row? the SECOND row? etc.

Obviously, such questions are haphazard. However, they do indicate the number and variety of quantitative situations that can occur daily, and the usefulness of the questions lies in the fact that they arise from natural, real-life situations.

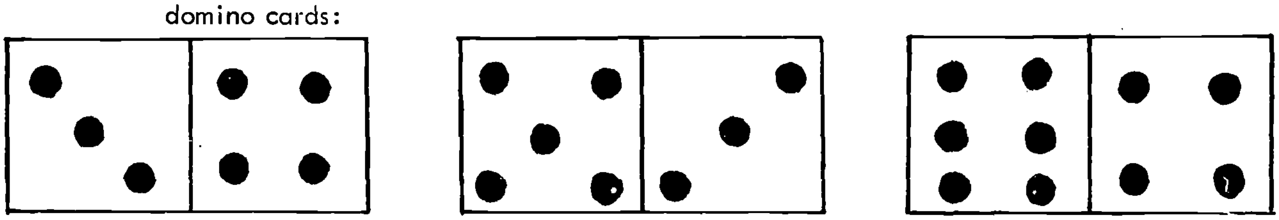
On a more orderly level, the following outline should be helpful to kindergarten teachers in giving them an overview of some of the important concepts tha. need to be dealt with at this level:

1. COUNTING

a) Identification of digits, numerals and their names
b) Counting by rote to at least 10. (cardinal numbers: 1, 2, 3, etc.)
c) Counting according to ORDER or ARRANGEMENT (ordinal numbers: lst, 2nd, 3rd, etc.)
d) Understanding "more than" and "less than" via counting.

2. QUANTITY

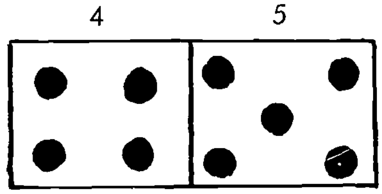
a) Understanding "more than" and "less than" by sight. A good aid for this is over-sized domino cards:



Each card can be treated individually. "Which 1/2 of the card has more dots? How many more?"

Numerals can be associated with each card as follows:

4 5



This can lay the foundation for number pairs and additon:

(4 , 5) 4 + 5 = 9

Also, the students will learn to recognize at a glance (without counting) grouping of numbers of objects and identify these with corresponding numerals.

b) Other quantitative concepts to develop are: big, little, larger than, smaller than, many, few.

3. MEASUREMENT

a) LINEAR: introduction to English units such as inch, foot, yard, mile. How tall? How far? How long?
b) VOLUME: cups, pints and half-pints (milk cartons), quart and gallon.
c) WEIGHTS: ounces and pounds; heavier than, lighter than, scales, balance, see-saw.
d) TIME: clock, hour, minute, seconds; yesterday, today, tomorrow; days, week, month, year; early, late; fast, slow; morning, noon, afternoon; seasons (time of year).
e) FRACTIONAL PARTS: 1/2, 1/4.
   (1) Groups: 1/2 of class; divide the jacks in half.
   (2) Part of One: fold the paper in half; in fourths.
f) TEMPERATURE: hot, cold, thermometer.

4. SETS OF GROUPINGS

a) A collection of objects.
b) One-to-one correspondence, matching.
c) Equal and equivalent sets.
d) Pairs (shoes, gloves, skates, drum sticks, eyes, ears, etc.)

5. MONEY

a) Names and values of coins.

6. POSITIONAL RELATIONSHIPS

a) In front of, behind; above, below; on top of, under; high, low; right, left.

7. GEOMETRY (SHAPES)

a) Form a circle to play.
b) What is "round"?
c) What is "square"?
d) Form a "triangle" with your fingers.

8. BASIC OPERATIONS (INFORMAL)

a) ADDITION: Use domino cards, number line, blocks, etc.
b) SUBTRACTION: Use domino cards. (How many dots should I "take away" on this half to have only 4 left?) Also, number line, blocks, songs, etc.

9. MISCELLANEOUS QUANTITATIVE SITUATIONS

a) Room numbers, table numbers, T.V. Channels, dozen (eggs), stamps (1st class, air mail), license tags, house and phone numbers, pages in books, chapter numbers, the number of swings and balls, the number right and the number wrong, and so on.

use many other opportunities to help children develop mathematical understandings. Obviously, informality is the key. Children should ENJOY their experience with numbers, and one of the best ways to bring this about is through the use of songs, games and attractive instructional aids. A source-book for these is the booklet "A SUGGESTED GUIDE FOR KINDERGARTEN TEACHERS" of Volusia County.

AN OVERVIEW OF THE CONTENT 1-6

CONCEPTS & ALGORITHMS	Grade I (160 pages)	Grade 2 (160 pages)
NUMBERS	Cardinals (To 99) Ordinals (1st - 6th)	Cardinals (To 999) Ordinals (To 9th)
WHOLE NUMBERS Add	1) All facts (thru 18) 2) 2 digit #'s: (7 + 20) (83 + 6) 3) Number line (10 + ?)	1) 3 digit #'s with carrying 2) 4 two-digit #'s 3) 3 rows, 1 to 3 digit #'s 4) Number line
Subtract	1) All facts (thru 18) 2) 2 digit #'s: (90 - 40) (78 - 5) 3) Number line	1) Re-teach facts 2) 3 digit #'s with borrowing: (133 - 75) 3) Number line
Multiply		1) All facts (thru 81) pg. 124-6 2) Number line
FRACTIONS	1/2 1/3 1/4	1/2 of 10; 1/3 of 12; 1/4 of 16; 2/3 of 12 and 1/2 = 2/4
PLACE VALUE	Ones, Tens	Ones, Tens, Hundreds
MEASUREMENT	Cups, oz., pts., qts.	Inch, foot, pints, qts., gals.
SHAPES (Geometry)		
TIME	Hours and 1/2 hours	Min., hrs., days, year Minutes on a clock Equivalent and non equivalent + , - , = , Numeral, Number name
MONEY	Pennies, nickels, dimes	Pennies to quarters
VOCABULARY (Used in Text)	Addition, Sum Subtraction, Difference, Minus Equation, Sentence Greater than Inverse Operations (do-undo) Multiples of ten Matching pair, number, numeral, number patterns, Numeration table, Place value	Points, Line segments, End points, models Open & closed geom. figures "right corners" (rt. angles) Rectangle, square, triangle, circle

CONCEPTS & ALGORITHMS	Grade 3 (305 pages)	Grade 4 (311 pages)
NUMBERS	1) Odd & Even Numbers 2) Multiples & primes	1) Common Factor & GCF 2) Modular arithmetic (+ , - , x) 3) Prime and Composite #'s 4) Number pairs (fractions) 5) Rational numbers 6) Base Ten
WHOLE NUMBERS Add	1) 3 #'s (67 + 58 + 54) 2) 4 #'s (347 + 6 + 38 + 52) Also: $16.27 + 3.86	1) 3, 4 & 5 digit addends to add. 2) Also: adding money
Subtract	1) 3 digit #'s, borrowing twice: (651 - 279) Also: $6.34 - 2.87; 8002 - 643 Checking (pg. 104)	1) Digit #'s: 1604 - 764; 6004 - 3775; 5000 - 3642 Checking.
Multiply	1) All tables (bk. 2) with re-teaching 2) 2 dgt. #'s x 2 dgt. #'s 2 dgt. # x 4 dgt. # "carrying" (short cut) Factor, factor, product	1) 3 digit #'s x 3 digit #'s (756 x 806) Also: 378 x 6 x 6
Divide	1) All facts thru 81 2) 1 dgt. divisor, 3 dgt. dividend Checking	1) 2 dgt. divisor, 3 dgt. dividend 2) 1 dgt. divisor, 4 dgt. dividend (Remainders shown as r9)
FRACTIONS	INFORMAL 1) Halves thru eighths 2) Equivalent Fractions	FORMAL 1) Improper fractions 2) Addition of fractions and mixed numerals 3) As number pairs 4) "Cross-product" method of checking fractions. 5) Lowest & highest terms
PLACE VALUE	Thousands, millions, billions, trillions Expanded notation	Thru trillions
MEASUREMENT	Linear, Area, Volume, "units" In., ft., yd., mi., cm., Rounding off, cups, pts., qts., gals.	Maintenance: Ditto Gr. 3

CONCEPTS & ALGORITHMS	Grade 3 (Cont'd)	Grade 4 (Cont'd)
GEOMETRY	Use of compass; circle, radius, diameter, chord, inside, outside. Angles & triangles: inside, outside, rt. angles and triangles quadrilaterals	Parallels, space Geometry, Central & Inscribed Angles Circles inscribed in triangles, Triangles inscribed in circles, Isosceles right triangle Parallelogram
SETS	Product sets, intersection	
SYMBOLS		
MONEY	Pennies to 1/2 dollars	
FUNCTION MACHINE	4 (RULE) 12	Ditto Gr. 3
VOCABULARY	Addends Factor, factor, product Order prin. for + & x Grouping prin. for + & x Quadrilaterals	Divisor, Quotient Improper fraction Mixed numeral Numerator, Denominator Perimeter, Diagonal Decade, Century, Millenium, Parallel lines Composite numbers Inscribed, circumscribed Rational numbers

CONCEPTS & ALGORITHMS	Grade 5 (314 pages)	Grade 6 (328 pages)
NUMBERS	1) Base Ten 2) Base Four (+ - x) 3) Roman numerals & others 4) Basic Principles 5) Number Pairs (3,7) 6) Factor trees 7) GCF LCM 8) Average	1) Maintenance 2) Bases Eight, Six, Four 3) Computing: Base Six 4) Prime factorization 5) Scientific notation 6) + & - Integers (Add and Subt.) Average (arith. mean)
WHOLE NUMBERS Add	Maintenance	Maintenance
Subtract	Maintenance	Maintenance
Multiply	Maintenance	Maintenance

CONCEPTS & ALGORITHMS	Grade 5 (Cont'd)	Grade 6 (Cont'd)
Divide	2 dgt. divisor, 5 dgt. dividend	3 dgt. divisor
FRACTIONS	1) Maintenance 2) LCM and LCD 3) Addition (Repeat) 4) Subtraction (in depth) 5) Multiplication 6) Division (informal) 7) Rational numbers	1) Maintenance 2) Inequalities
DECIMALS	1) Reading and writing 2) Add - Subtract 3) Metric System	1) Maintenance 2) Rounding 3) Multiplying 4) Dividing
PER CENT		1) Fraction-Decimal Equivalents 2) Problems: Standard types
EXPONENTS		Used with Place Value
PLACE VALUE	Maintenance	Maintenance Exponents
MEASUREMENT	1) Maintenance 2) Protractor, degrees 3) Metric System (linear) in depth.	1) Maintenance: area, surface area, volume 2) Radians, diameter, circum., area of a circle 3) Metric System: Linear, weights and capacity
RATIO	Also: Scale Drawing	Maintenance
GEOMETRY	1) More abstraction: rays, planes 2) Congruent segments, $\angle$'s, and $\triangle$'s. 3) Constructions 4) 3 dimensional figures	1) Vertices 2) Congruent angles & triangles 3) Space geometry 4) Cross sections of 3 dimensional figures 5) Area of triangles 6) Ratio and similar triangles 7) Pythagorean Theorem
GRAPHING	On lines & planes (formal)	Rational numbers, functions, + & - integers
SETS	1) Union, Intersection 2) Solution sets 3) Sets of ordered pairs	1) Maintenance 2) Venn Diagrams

DENOMINATE NOS.	Converting units	Maintenance
FUNCTION MACHINE	$f(2) = 4$	$f(n) = 5 \times 10^3 \times n$
ESTIMATION		
FORMULAS	1) $A = L \times W$ 2) $d = r \times t$ 3) $v = L \times w \times w$	1) $C = \pi \times d$ $A = \pi \times r^2$
VOCABULARY	Reduce, LCD & LCM Commutative Associative Distributive Cubic Inches Cubic cm., liters Congruent segments Sphere, rect., pyramid, rect. prism, cylinder, cone, Union and intersection	dimensions expanded notation prime factorization radian vertex isosceles perpendicular bisector straight angles polygons similar triangles Pythagorean Theorem Integers

NOTE: Each book company publishes a scope and sequence chart which can be used to supplement above. They will be sent free on request.

SILVER BURDETT SERIES

CONCEPTS & ALGORITHMS	Grade 1 (142 pages)	Grade 2 (144 pages)
NUMBERS	Cardinals (To 150) Ordinals (Thru 9th) Counting by 10's to 90 Matching one to one Number line	Counting by 2's; 5's; 10's Writing numerals to 200
WHOLE NUMBERS		
Add	Facts thru 10 56 + 31 = ? 2 + ? + 7 2 + 2 + 3 + ? Money	All facts thru 18 40 +83 85 - ___ = 32 34 + ___ = 79
Subtract	97 -62 6 - ? = 4	All facts thru 18 109 -66
Multiply		"Nine 2's is ____?"
Divide		"How many 4's in 16?"
FRACTIONS	1/2 , 1/3	Maintenance; 1/4
PLACE VALUE	To hundreds (150) using bundles	382 means ___ ___ ___ (H T O)
MEASUREMENT	Pints and quarts; 1/2 pints Inches	Maintenance Temperature, degrees
SHAPES		
TIME	Hours, 1/2 hours	Quarter hours
SETS	To compare numbers	
SYMBOLS		+ - and degrees
MONEY	Pennies, nickels, dimes	Quarters, halves
VOCABULARY	Number, numeral Largest, smallest Number Line Number sentences	Addition, subtraction Temperature Increase, decrease

CONCEPTS & ALGORITHMS	Grade 3 (292 pages)	Grade 4 (292 pages)
NUMBERS	Patterns, even, odd re-naming, sentences, Counting to 1000 Numbers & numerals to 99,999 Roman numerals Rounding, expanded notation (Renaming numbers)	Sets of factors, common factors; associative, commutative, distributive principles; Identity element, expanded notation (renaming numbers), Roman numerals to hundreds
WHOLE NUMBERS Add	Use of Frames 3 columns, 3 rows	Maintenance 2 columns, 4 rows
Subtract	By expanded notation: 642 − 589 Checking Opposite to adding	703 − 346; 92,232 − 45,798
Multiply	Facts thru 9's (p. 272) 1 digit multipliers	2 digit multipliers, 2 digit multiplicands
Divide	1 digit divisors Opposite to multiplying Tabular method only	2 digit divisors, 4 digit dividends Division "undoes" multiplication
FRACTIONS Add Subtract	1/4's, 1/5's, 1/6's	Equivalent fractions With like denominators With like denominators: (1 1/8 - 3/8) Improper fractions to mixed numerals & back, 2/4 of 16
PLACE VALUE	To ten thousands place	Through millions Value of each place
MEASUREMENT	Cups, pints, quarts, gallons; freezing point; pound, ounce; miles, feet, to the nearest inch, 1/2 in. and 1/4 in.	Maintenance Meaning of a Unit of measure. Ounces in a cup, pint, quart. Dry measure: pint, quart, peck, bushel. Tons, time to seconds; Scale drawing
GEOMETRY	Simple closed curves, points, rectangle, square, circle, line segments, cubes, spheres	Maintenance Rays, angles, endpoints, right triangles, cubes
GRAPHING	Picture graph	Making bar graphs; reading and making line graphs
TIME	Minutes	Seconds

CONCEPTS & ALGORITHMS	Grade 3 (Cont'd)	Grade 4 (Cont'd)
SETS	Braces	Maintenance
SYMBOLS	$< > + - \times \neq$	Maintenance $\{ \}$ and degree°
VOCABULARY	Addition, subtraction, multiplication, division, quotient, remainder, minus sign, difference, parentheses, collection, geometry, numeral, sentences	Maintenance Equivalent (fractions) Identity elements for + and x Numerator, denominator Congruent, common factors, commutative, associative, distributive, radius, diameter, congruent

CONCEPTS & ALGORITHMS	Grade 5 (306 pages)	Grade 6 (305 pages)
NUMBERS	Roman numerals to thousands. Base five: counting and place value Changing base five to base ten Rounding off to nearest 10 and 100 Primes, composites, multiples odd, even, prime factor, greatest common factor (g.c.f.) least common multiple (l.c.m.)	Cardinal number (number of the set), subset, empty set, partition of sets, periods to billions, expanded notation, exponents: $(9 \times 10^0) + (4 \times 10^2) + 5 = N$ Directed numbers: $1/4 + (-2) = N$ and $N + (-1/2) = 1\ 1/2$ Base six; changing Base ten to Base six; prime trees, prime factorization, closure
WHOLE NUMBERS	Maintenance	Maintenance
Add		Maintenance
Subtract	Inverse of addition Using expanded notation	$\begin{array}{r} 247,005 \\ \underline{-168,406} \end{array}$
Multiply	$\begin{array}{r} 618 \\ \underline{\times 72} \end{array}$ 700 x 483	$\begin{array}{r} 867 \\ \underline{\times 870} \end{array}$ $\begin{array}{r} 439 \\ \underline{\times 306} \end{array}$
Divide	$78\overline{)3,699}$	$6025\overline{)7,118,422}$
FRACTIONS	Changing mixed fractions to improper fractions; proper fractions; lowest terms; like fractions	Maintenance Common denominator; least common denominator

CONCEPTS & ALGORITHMS	Grade 5 (Cont'd)	Grade 6 (Cont'd)
Add	5 1/2 + 3 1/4 + 6 2/3	Maintenance
Subtract	12 7/16 - 6 7/8	Maintenance
Multiply	$\frac{5}{8} \times \frac{2}{5}$; $4 \times \frac{3}{8}$	Full treatment
Divide	$\left.\begin{array}{l} \frac{5}{9} \times 9 = 5 \\ 5 \div 9 = \square \end{array}\right\}$	Full treatment; common denominator method; complex fractions
DECIMALS	Decimal point; decimal fraction; decimal mixed fraction Reading and writing to thousandths. Add: 4 rows Subtract: 13.045 - 7.266 Multiply: 143.6 x1.41	Reading and writing decimals; Rounding; +, -, x, ÷ Fraction-decimal equivalents
PER CENT		Per cents, ratios and fractions, Fraction, decimal, per cent equivalents. Interest, discount. The 3 standard forms of: Base x rate = percentage % of increase or decrease
PLACE VALUE	Expanded notation to millions Exponents	Maintenance
MEASUREMENT	Change 1 pk. 3 qt. to qts. To nearest 1/2 in. and 1/4 in. Scale drawing; standard Unit of Measure; temperature (from -10F to 32F); METRIC SYSTEM: millimeter, centimeter, decimeter, meter. Add and subtract denominate #'s.	Maintenance Precision, error of measurement, greatest possible error. +, -, x, ÷ denominate #'s Metric System (Linear) and English equivalents. Prisms, faces, pentagons, lateral area, cylinders, cones, pyramids, scalene, isosceles, equilateral triangles
RATIO	Comparing numbers; equivalent, ratios; number pairs having same ratio	Ratio maintenance and proportion; scale drawing; map reading

CONCEPTS & ALGORITHMS	Grade 5 (Cont'd)	Grade 6 (Cont'd)
GEOMETRY	Point of intersection; line segment; endpoints; congruency; vertex; angles; interior and exterior of angles; rays; degree measure; right angle; perpendiculars; parallel lines; plane figure; simple closed figures; polygons; quadrilaterals; rectangle; squares; diagonals; regions; areas; square feet and yards; cubes; rectangular prisms; edges; radius, diameter, arcs, arc degrees; sphere; hemisphere	Perimeter, rectangular regions; square inches, sq. ft., sq. yd.; Maintenance: points, line segments, endpoints, etc. Union and intersection. Right angles and triangles Intersection of planes -- a line Maintenance: rectangles, cubes, circles, spheres, etc. Prisms, rectangular prisms, lateral faces, pentagonal prisms, cylinders, scalene, equilateral, isosceles.
GRAPHING	Bar, line and pictograph. Ordered pairs; using ratios and graphs	Number pairs, ordered pairs. Divided bar graphs; circle graphs
TIME	Standard time zones. Latitude; longitude; meridian; prime meridian	
VOCABULARY	Inverse, inequality, equation, partial products, commutative, associative, distributive, identity element, array, ratio. (Also: see GEOMETRY and MEASUREMENT.)	Maintenance Factorization; closure; proportions, means, extremes. Union and Intersection. Prisms, pentagonal, cylinders, scalene, isosceles, equilateral, (Also: see GEOMETRY and MEASUREMENT.)

GENERAL MATHEMATICAL CONTENT

GRADES 7-9

At the elementary level students are generally in self-contained classrooms. In such situati the teacher is responsible for differentiating instruction to meet the wide range of abilities in his cl At the junior high school level, however, there are at least 3 levels of mathematics which help narı the range of abilities, enabling teachers to better meet the needs of all students. Aiding teachers i this regard was the adoption of multiple texts for use at a given grade level. Currently, texts are available for slow, average and fast groups for levels 7 through 9.

What curriculum should prevail for each group at each grade level? Rather than catalog pa after page of detailed information that may be rarely referred to, if ever, it seems that it would be more sensible to have our state-adopted texts serve as basic curriculum "guides". Since no single te book can fully meet the needs of all students in any one class, the teacher ould then be responsibl for supplementing the text material to whatever degree necessary to give ximum help to each stud If the teacher wishes, he can secure a chart indicating the major concepts dealt with in each text b contacting the publisher or his representative. Frequently, the teacher's edition of the text will co tain this and other useful information. You may receive a free copy of a teacher's edition with your class-sized order for books by requesting one with your order.

In addition to the above, each school should develop supplementary guidelines to fit their individual school situations. While multiple adoptions are of some benefit, there is still a pressing need to develop special materials and procedures for dealing effectively with the needs of slow lear in mathematics.

GRADES 10-12

At these grade levels, students generally have but 2 choices: the traditional mathematics offerings of algebra, geometry, trigonometry, and "fifth year mathematics" or the general math type course. Here again the texts themselves determine, in the main, the content of the course, and therefore serve primarily as curriculum guides. These, of course, are supplemented by individual teacher initiative, state developed curriculum guides, and the document you are now reading. Each school should have its own curriculum guide developed by the mathematics faculty. This guide should be periodically revised as needed.

GRADES SHOULD NOT DEPEND UPON TEST SCORES ALONE.

EVALUATION

The evaluation of the mathematics program should be a continuous process devoted to all aspects of the program, including pupil achievement. Some ways to do this include:

1) Teacher observation
2) Standard achievement tests
3) Textbook and teacher-made tests, and short daily quizzes
4) Teacher-pupil conferences
5) Homework

TEACHER OBSERVATION

Much can be learned about a student by simply observing his performance in class. Does he have a healthy attitude toward his work and his teacher? Is he willing to cooperate with his teacher for his own educational benefit? Does he hinder or help the progress of the class during the course of the school year? Does he do neat or sloppy work? All of these questions are pertinent to the overall evaluation of the student. However, academic grades should not be cut due to misbehavior.

STANDARD ACHIEVEMENT TESTS

A great deal of time and expense is expended in giving and scoring standardized tests. They provide valuable information regarding a student's past performance and his potential to perform. His language and non-language I.Q., and his scores in reading, mathematics, and other areas give the teacher some of the details needed for a complete academic portrait of each student. The teacher should seek this information as needed.

TEXTBOOK AND TEACHER-MADE TESTS

Most textbook companies sell tests which accompany their textbooks. These should be examined carefully to be sure all items are pertinent to what has been covered in class. If some are not sufficiently clear or pertinent, they should be excluded. Also, the NCTM publishes a booklet.

The general consensus of experienced teachers is that students should be quizzed and tested regularly. The nature of such quizzes and tests would depend upon the grade level. Oral quizzes or "recitations skillfully given, can be very effective. In this way the teacher can probe each student's comprehensi of a topic through various avenues. Such quizzes could well serve as a review prior to giving a forma test. It may save time and frustration should the teacher discover that students are not quite as ready for the test as they should be.

Experienced teachers also recommend 10 or 15 minute written quizzes which may be checked quickly in class. These tend to keep students "primed" and also tend to discourage last minute cramming for more formal tests.

In preparing formal tests, care should be taken to be sure that most students can finish the test in the time allotted. Since the primary goal in mathematics classes is to help students learn to solve written problems, it is important, that, where pertinent, such problems be included on tests. In general, a minimum of 3 formal tests should be given during a six weeks period. The day before each test is given students should be told precisely what material the test will cover and should be given some idea of the general format of the test to help alleviate some of the apprehension normally associated with test-taking. For instance, they might be told the number of items, how they are "weighted" or the number of problems devoted to certain topics.

A teacher-made test should serve at least 3 major purposes. First, it should serve to evaluate the student's understanding of the material covered. Secondly, it should serve as an indication to the teacher as to how effective a teaching job was done. And finally, it should serve as a teaching device whereby each item is thoroughly reviewed and understood by all students. Where the general performance of students is below the expected level, the teacher should re-examine the test and his presentation of the material. If, in his judgment, one or both of these aspects were inadequate, the students should be reviewed and re-tested.

In general, tests should be corrected and returned to students within 2 to 3 days while the material is fresh in their minds. Care should be taken not to scribble such words as "careless error" where minor errors occur. Conscientious students DO care, and like some of us adults, they do occasionally make an error, which is sufficient punishment in itself.

TEACHER-PUPIL CONFERENCES

Teacher-pupil conferences should be held as often as practical. All of us appreciate personal attention and to those students who may get little of it outside of school, the teacher's overt, personal interest in their individual problems may be just what it takes to improve their self-images and their school work. In addition, the reasons for a student's poor performance on a test, for example, may not be apparent.

However, by asking a student to explain verbally how he did the problem may clear up the difficulty quickly and permanently. A case in point is the student who apparently couldn't "add". When he added orally for his teacher, however, it turned out he could add. His problem was he didn't know to regroup ("carry").

HOMEWORK

When homework is given, it should be corrected and students should be given credit for having done it. While it may be true that a few students might not do it themselves, there can be little justification for assigning work for which no credit is to be given. In assigning homework the teacher assumes an obligation for seeing to it that the work is checked and corrected. This is particularly important since all of the homework may have been done incorrectly. On occasion, some homework-type assignments should be done in class where the teacher can help and encourage students in their work.

EARLY DIAGNOSIS

During the first week of school teachers will find it beneficial to give students a self-made or published diagnostic test. This will check each student's background and preparedness for the classes they are in. A few sample tests follow, which you may duplicate, if you wish:

ADDITION (40 Items, 2 Minutes)

4	3	6	2	2	4	5	5
+2	+5	+3	+7	+3	+3	+2	+5
3	2	4	7	4	9	5	5
+3	+6	+4	+3	+5	+1	+6	+0
8	9	7	6	5	2	0	7
+3	+5	+5	+6	+8	+9	+8	+6
6	9	0	6	9	7	8	2
+9	+6	+7	+4	+9	+7	+7	+8
8	7	8	3	8	4	7	3
+8	+4	+6	+0	+9	+8	+9	+9

SUBTRACTION (40 Items, 2 Minutes)

6	7	8	6	9	6	9	8
-2	-3	-5	-0	-3	-3	-6	-3
9	8	7	9	6	9	8	8
-5	-1	-4	-4	-4	-2	-4	-2
12	16	14	13	10	16	17	15
- 6	- 8	- 9	- 8	- 4	- 7	- 9	- 8
12	15	11	9	13	16	17	13
- 5	- 7	- 4	-7	- 5	- 9	- 8	- 7
10	14	12	13	11	14	13	18
- 5	- 6	- 7	- 6	- 6	- 8	- 4	- 9

END OF TEST

DIAGNOSTIC TEST ON MULTIPLICATION AND DIVISION COMBINATIONS

MULTIPLICATION (32 Items, 2 Minutes)

$\begin{array}{r}2\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}2\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}4\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}6\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}5\\ \times 4\\ \hline\end{array}$
$\begin{array}{r}2\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}5\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}4\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}6\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}2\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}9\\ \times 2\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 8\\ \hline\end{array}$
$\begin{array}{r}4\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}3\\ \times 9\\ \hline\end{array}$	$\begin{array}{r}6\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}4\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}5\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}5\\ \times 9\\ \hline\end{array}$	$\begin{array}{r}7\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}8\\ \times 5\\ \hline\end{array}$
$\begin{array}{r}7\\ \times 9\\ \hline\end{array}$	$\begin{array}{r}9\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}6\\ \times 9\\ \hline\end{array}$	$\begin{array}{r}11\\ \times 12\\ \hline\end{array}$	$\begin{array}{r}9\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}11\\ \times 11\\ \hline\end{array}$	$\begin{array}{r}12\\ \times 12\\ \hline\end{array}$	$\begin{array}{r}9\\ \times 9\\ \hline\end{array}$

DIVISION (56 Items, 3 Minutes)

$2\overline{)14}$	$3\overline{)18}$	$2\overline{)16}$	$3\overline{)24}$	$2\overline{)12}$	$3\overline{)15}$	$2\overline{)18}$	$3\overline{)12}$
$4\overline{)24}$	$2\overline{)18}$	$3\overline{)21}$	$2\overline{)10}$	$3\overline{)27}$	$4\overline{)32}$	$4\overline{)20}$	$4\overline{)12}$
$5\overline{)20}$	$5\overline{)30}$	$4\overline{)12}$	$4\overline{)16}$	$5\overline{)40}$	$6\overline{)42}$	$5\overline{)15}$	$4\overline{)36}$
$5\overline{)35}$	$5\overline{)25}$	$6\overline{)18}$	$6\overline{)48}$	$6\overline{)24}$	$6\overline{)36}$	$6\overline{)30}$	$6\overline{)54}$
$7\overline{)14}$	$7\overline{)49}$	$7\overline{)28}$	$7\overline{)63}$	$7\overline{)42}$	$7\overline{)21}$	$7\overline{)56}$	$7\overline{)35}$
$9\overline{)63}$	$8\overline{)32}$	$9\overline{)54}$	$8\overline{)24}$	$9\overline{)81}$	$8\overline{)56}$	$9\overline{)18}$	$9\overline{)36}$
$8\overline{)72}$	$9\overline{)27}$	$8\overline{)40}$	$9\overline{)45}$	$8\overline{)16}$	$9\overline{)72}$	$8\overline{)48}$	$8\overline{)64}$

END OF TEST

THIS TEST WILL NOT COUNT TOWARD YOUR REGULAR GRADE.

GENERAL ACHIEVEMENT TEST

DO NOT WRITE ON THIS TEST

PART I - WHOLE NUMBERS

DO THE FOLLOWING:

1)
$$\begin{array}{r} 586 \\ 79 \\ 857 \\ 65 \\ +\ 1998 \\ \hline \end{array}$$

(2) 39 + 9 + 307 + 880

(3) 84 + 900 + 67 + 8

(4) 14,009 + 1,468 + 372 + 47

(5)
$$\begin{array}{r} 58,379 \\ 8,076 \\ +\ 7,997 \\ \hline \end{array}$$

6) Subtract 694 from 1,242

7) Find the difference between 5,622 and 5,837

8) How much larger is 10,080 than 6,288?

9) 11,000 minus 1,186

10) 2004 - 186

11) 7,865 x 94

12) 680 x 75,000

13) Find the product of 64,007 and 305

14) Multiply 367 by 3,008

15) What is 605 times 463?

IN DOING THE FOLLOWING, SHOW REMAINDERS USING A SMALL "r":

16) 3,008 ÷ 68

17) Divide 4,325 by 302

18) $\frac{7,503}{79}$

19) $540\overline{)541,371}$

20) 693,002 ÷ 78

PART 2 FRACTIONS

IN DOING THE FOLLOWING REDUCE ALL FRACTIONS TO LOWEST TERMS:

1) Find the sum of $3\frac{3}{4}$ and $40\frac{7}{12}$

2) Add the sum of $7\frac{5}{6}$ and $6\frac{3}{5}$ to 10

3) $\frac{3}{4} + \frac{4}{5} + \frac{2}{3}$

4) $10\frac{1}{4} + 8\frac{7}{9} + \frac{2}{3}$

5) $\frac{1}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$

6) Subtract $3\frac{13}{18}$ from $3\frac{5}{6}$

7) Take $33\frac{5}{7}$ from $34\frac{1}{6}$

8) $70\frac{5}{7}$ minus $3\frac{5}{6}$

9) $100 - 10\frac{5}{7}$

10) $50\frac{2}{9} - 27$

11) Find the product of $25\frac{5}{8}$ and $3\frac{5}{6}$

12) Multiply $12\frac{2}{3}$ by $1\frac{8}{19}$

13) $\frac{5}{6} \times \frac{3}{7}$

14) $6\frac{2}{3} \times 3\frac{1}{3}$

15) $3\frac{1}{8} \times \frac{4}{25} \times 33\frac{1}{3}$

16) Divide $\frac{7}{12}$ by $2\frac{1}{3}$

17) $9 \div \frac{1}{9}$

18) $3\frac{1}{3} \div 6\frac{2}{3}$

19) $\frac{7}{10} \div 10$

20) $\dfrac{8}{\frac{1}{4}}$

PART 3 DECIMALS

WRITE THE FOLLOWING IN DECIMAL FORM USING NUMERALS:

1) Three and one hundred four, ten-thousandths.

2) One thousand forty-seven, hundred-thousandths.

3) Four thousand two hundred seventy-three, millionths

4) Six hundred and six thousandths

5) Thirty-five thousandths

6) Six and two hundredths

CHANGE THE FOLLOWING TO DECIMALS: (Round off to hundredths.)

7) $\frac{5}{8}$ 8) $\frac{2}{7}$ 9) $\frac{1}{6}$

CHANGE THE FOLLOWING TO DECIMALS: (Round off to thousandths.)

10) $\frac{8}{15}$ 11) $\frac{11}{12}$ 12) $\frac{3}{104}$

DO THE FOLLOWING:

13) $3.04 + .8 + 70.003 + .0097$

14) Subtract the smaller from the larger: .0097 ; .04

15) $1.3104 \div 42$

16) $30.004 \times .025$

17) $.00144 \div .012$

18) $48 \div .0003$

CHANGE THE FOLLOWING TO COMMON FRACTIONS REDUCED TO LOWEST TERMS:

19) .375 20) .075

PART 4 PER CENT

CHANGE THE FOLLOWING TO PER CENT:

1) $\frac{5}{8}$ 2) $\frac{1}{50}$ 3) 66 4) $\frac{1}{100}$ 5) $\frac{3}{25}$

6) .125 7) .025 8) .001 9) .5 10) 5

CHANGE THE FOLLOWING TO FRACTIONS REDUCED TO LOWEST TERMS, OR TO WHOLE NUMBERS:

11) 1%　　12) $33\frac{1}{3}\%$　　13) 300%　　14) $\frac{1}{2}\%$　　15) 10%

FIND:

16) 2% of 150　　17) $\frac{1}{5}\%$ of 25　　18) 30% of 1200

19) 400% of 5　　20) 10% of 100

GENERAL ACHIEVEMENT TEST

(AVERAGE TIME: 2 HOURS)

TO THE STUDENT:

Do all your work neatly on your own paper. As you arrive at an answer, place it next to the proper number below. After completing this test, place it on top of your work sheets and turn them in to your teacher.

STUDENT'S NAME ____________ SCHOOL ____________ DATE ________

PART 1 WHOLE NUMBERS	PART 2 FRACTIONS	PART 3 DECIMALS	PART 4 PER CENT
1) 3,585	1) 44 1/3	1) 3.0104	1) 62 1/2%
2) 1,235	2) 24 13/30	2) .01047	2) 2%
3) 1,059	3) 2 13/60	3) .004273	3) 6600%
4) 15,896	4) 19 25/36	4) 600.006	4) 1%
5) 74,452	5) 2 43/60	5) .035	5) 12%
6) 548	6) 1/9	6) 6.02	6) 12.5%
7) 215	7) 19/42	7) .63	7) 2.5%
8) 3,792	8) 66 37/42	8) .29	8) .1%
9) 9,814	9) 89 2/7	9) .17	9) 50%
10) 1,818	10) 23 2/9	10) .533	10) 500%
11) 739,310	11) 98 11/48	11) .917	11) 1/100
12) 51,000,000	12) 18	12) .029	12) 1/3
13) 19,522,135	13) 5/14	13) 73.8527	13) 3
14) 1,103,936	14) 22 2/9	14) .0303	14) 1/200
15) 280,115	15) 16 2/3	15) .0312	15) 1/10
16) 44 r 16	16) 1/4	16) .7501	16) 3
17) 14 r 97	17) 81	17) .12	17) .05
18) 94 r 77	18) 1/2	18) 160,000	18) 360
19) 1002 r 291	19) 7/100	19) 3/8	19) 20
20) 8884 r 50	20) 32	20) 3/40	20) 10

SOME SUGGESTIONS FOR HELPING STUDENTS SOLVE WORD PROBLEMS

Once a student has mastered an algorithm, he is well on his way toward taking the next logical ste which is to solve a practical problem necessitating the use of the algorithm. Hence, after careful study and full understanding of the algorithm related to the division of decimals, he should be ready analyze the following word problem and render the correct solution:

> If gold can be hammered into sheets 0.000005 of an inch thick, how many sheets would it take to make a stack 0.5 inch high?

Assuming a student knows the meaning of all of the words in the problem, his next task is to determi what quantities are involved and how they are related. This is the most critical step in the entire process. The fact that he has mastered the algorithm won't help him here. This is a THINK situati

What am I asked to find?	How do I find it?
Do I multiply or divide?	WHAT is to be multiplied or divided by what?

Some students may readily sense what must be done to solve such a problem. Most students, howeve will need some guidance to help them see that this problem is one of many which follow the general pattern:

THE NUMBER OF THINGS X SIZE OF EACH = A TOTAL

If, prior to this, the student was taught to list "what is given", he will usually have little trouble solving the problem:

GIVEN:

SIZE OF EACH ITEM: 0.000005 in.
TOTAL: 0.5 in.
NUMBER OF ITEMS: N

SOLUTION: (Unfinished)

NUMBER X SIZE = TOTAL
N X .000005 = .5

Assuming that a student understands that a product divided by one of its factors gives the other, the operation involved is obvious. The same pattern ("formula") can be used to solve problems involving such things as tickets and the cost of each, and many others.

Problems related to the one used above are the ones which students deal with most frequently. However, on occasion, students should be asked to think through a problem such as the following:

How can we find the perimeter of this desk top?

Unfortunately, students are too often told or shown how to solve such problems in an unstimulating, prosaic manner: "Boys and girls, add the lengths of the four sides, and you've got it!

However, if students are given the chance to think for themselves, they would probably offer several methods like the following:

1. Measure the length, and double it; measure the width, and double it. Adding the two results gives the perimeter.
2. Find the sum of a length and a width and double it.
3. Run a tape measure all around the desk top.
4. If no tape measure is available, run a string around the desk top and measure the string with a ruler or yardstick.

Similar exercises in creative problem solving would involve such questions as:

1. How can we find the AREA of the desk top?
2. A BEE is sitting on a line drawn on the chalkboard. How can we locate its exact positio
3. A BEE is sitting on a point on the chalkboard having no lines drawn on it. How can we pinpoint its location on the plane?
4. How can we pinpoint the location of a spider hanging from the ceiling?
5. How can we determine the number of 1 inch cubes in a stack measuring 3 in. x 3 in. x 3 ir

Developing skill in solving word problems necessitates the same basic requirement needed to develop any skill, and that basic requirement is PRACTICE. The teacher must carefully apportion his time to be sure that ample time is allotted for this most important aspect of mathematics instruction.

One of the many important tasks of the teacher is to simplify the complicated, where possible. For example, many texts will use several forms of a simple formula where only one form is needed. An illustration of this is the way DISTANCE, RATE, and TIME problems are treated. We often see the following three arrangements:

$d = r \times t$ $\qquad\qquad \frac{d}{r} = t$ $\qquad\qquad \frac{d}{t} = r$

Since most students find problem solving difficult anyway, why not simply use the first arrangement for all such problems? If a student lists what is given and applies this information to the one formula, he will be able to see at a glance what must be done to solve the problem if he understands the relationshi between 2 factors and their product:

$d = r \times t$	$d = r \times t$	$d = r \times t$
$d = 50 \times 3$	$150 = r \times 3$	$150 = 50 \times t$

The same thinking can be applied to $A = L \times W$, $C = \pi \times d$, $A = \pi \times r^2$, and similar formulas.

We must provide our students the training and help they need to acquire the unique habits needed for independent study in mathematics. We all need to encourage and expect students to READ THEIR TEXTBOOKS and not consider the text as simply a source of problems to be solved. Too many students complete their formal education believing they cannot learn mathematics by reading. Some suggestions for teachers to pass on to their students are:

1. READ CAREFULLY AND DELIBERATELY: Do not expect to fully understand a discussion or a problem after the first reading. Unlike fiction, reading mathematics requires a high degree of concentration.

2. THINK WITH A PENCIL AND SCRATCH PAPER: Try to anwer questions and solve problems before reading the solution. Watching someone else do something is not nearly as helpful to you as doing it yourself.

3. BE INDEPENDENT: Don't make a habit of using your teacher or others as a crutch. However, after an honest try, it would be foolish not to seek any help you may need. It is academically dangerous to allow yourself to get behind in mathematics since much of what you learn is based upon previous work.

4. LISTEN CAREFULLY AND TAKE GOOD NOTES: Tests have shown that most students can recall very little of what they hear during a 1 hour lecture. In addition, most mathematics texts rely upon teachers to explain the various topics in far more detail than is possible in a single text. Hence, failure to take full advantage of a teacher's carefully prepared presentation can cause serious academic difficulties.

5. LEARN TO BE PERSISTENT: Resist, as much as possible, the temptation to take the easy way out. There is much satisfaction to be gained from working out problems on your own, not only in math but in other areas as well. If you reach an impasse, take a break or go on to something else for a while. A fresh start or a new approach often does the trick.

6. BE NEAT AND ALWAYS DOUBLE-CHECK YOUR WORK IF TIME PERMITS: Sloppy work and "careless errors" are close friends. Being neat will make it easier for you to re-trace your steps and locate possible errors. Experienced carpenters use this rule: Always measure twice! Follow this rule when you take exams; always do problems twice when time permits. If you have trouble lining up your work, use graph paper.

7. DON'T JUMP INTO A PROBLEM BEFORE YOU'VE LOOKED IT OVER CAREFULLY: Make it a habit to read a problem through a couple of times before digging in. Many times a labelled sketch will help. List what is given, concentrate on the question to be answered, and then pursue the solution.

8. PLACE PROBLEMS ON 3" x 5" CARDS FOR FUTURE REFERENCE: A good way to keep current in mathematics is to use one side of a 3" x 5" file card to state a problem, and the other side for the solution. These will be most useful prior to taking tests, especially if a portable or wall chalkboard is available. In math, as in most other work, "practice makes perfect". The more problems you do, the easier they become.

THE "SLOW" STUDENT IN MATHEMATICS

". . . the race is not to the swift . . ."
Ecclesiastes 9:11

How many people can run a mile in 4 minutes? in 5? In all probability, there are many people who cannot run a mile at all! Is this a disgrace? What about slow swimmers, slow seamstresses and slow artists? Is slowness a synonym for stupidity? Unfortunately, this is precisely the attitude often projected toward those who happen to be slow in mathematics.

The following appeared in the March, 1965, issue of the MATHEMATICS TEACHER in an article by K. Sassé entitled "Mathematics for the Non-college bound in Junior High School":

> "Parenthelically, it is interesting to meditate upon Dr. Earnest O. Melby's observation that the teaching profession stands alone in that its members wish to avoid those who need them most. In medicine, the physician deals almost exclusively with the sick; and the more esoteric the illness the more professional interest he has in it."

And, Dr. Donovan Johnson, past president of the NCTM in a speech entitled "Looking at Math from the Student's Point of View" urged that all students be given an opportunity to savor success in mathematics however relative.

Frustration, he indicated, usually results in a life-long antipathy toward math. He also outlined the negative reactions of students as revealed by a university-conducted study. These were: lack of understanding and success; math's lack of application to life situations; boring drill; impatient teachers.

Then, we all need to reflect upon this quote from John Holt's article in GRADE TEACHER, March, 1966:

> "----strategies used by less successful students to avoid the humiliation of trying and failing. If you expect to be wrong, if you don't think that you're going to succeed, the way to protect your self-esteem -- whatever little of it you have left, your vanity, you ego -- is not to try. You can't be ashamed, if you know in advance that you didn't try. You can't be humiliated before the class if you make it clear to everybody that you didn't try. So the teacher asks a question and you giggle and clown. It's perfectly obvious that you're not trying. So you get the wrong answer, but everybody knows you weren't trying. Whether you could have gotten the right answer or not remains a mystery. Nobody knows. You've spared yourself the humiliation of trying and failing. This is a very popular strategy in the classroom."

Where do we go from here? One of the first things we need to consider are the GOALS relating to slow learners in mathematics. In a previous section, it was stated that the ultimate goal in education should be the development of a worthwhile citizen who can make a positive contribution to society. What are the academic requirements for slow learners in this regard?

Essentially, the slow learner should be able to understand the kind of mathematics used in newspapers and daily living. This would include items such as budgets (personal, city, state and national), voting (what constitutes a simple majority and a 2/3 majority), money (prices, discounts, interest rates), averages (rainfall, salaries, baseball), and sizes and quantities (square yards of rug, square inches on a T.V. screen, capacities of refrigerators). Of course, in addition to these understandings, he should be a proficient as possible in the fundamentals of arithmetical computations.

Is this all that slow mathematics students should know? Not at all. These are but the basics. Some history, some geometry, games, puzzles and so on all have a role to play in providing some depth of understanding to the nature of mathematics in general. Also, these activities will provide the kind of variety any class needs to make it more interesting.

HELPING THE SLOW LEARNER IN MATHEMATICS

"Repeated failure destroys the personality of the learner." John R. Clark
A.T., Dec., 1965

Trying to teach mathematics in heterogeneous classes using a single text creates formidable problems for both students and teachers. Many schools tend toward grouping students who are slow in mathem so that teachers can make better provisions for dealing with THEIR problems than is possible in mixe classes. However, it must be emphasized strongly that such grouping is worthless unless there is an adjustment in curriculum, materials, methods, attitudes and expectations. The tell-drill-test approa is the least apt to succeed with these youngsters. The year should start with a fresh approach insteac of bombarding the students with the same old stuff they have been failing for years. Games, puzzle: interesting flash cards, (see Appendix, page 74), laboratory type activities and some opportunity to work individually on simple projects will motivate these students far more effectively than lectures, exhortations, and threats. Of course, such classes should be kept as small as possible, and the most creative, enthusiastic and emphathetic teachers should be encouraged to teach at least one such clas Too often such classes are assigned to new teachers who lack the experience and confidence required to help these students.

HELPING THE SLOW STUDENT IN MATHEMATICS READ MORE PROFICIENTLY

Since the inability to read well is probably the most serious problem affecting the progress of the slov learners in math, a great deal of emphasis must be placed upon this aspect of any program designed t help such students. "What? Teach reading? But I'm a MATH teacher! I don't know anything abc teaching READING?"

While it is true that certain aspects of teaching reading should be left to those trained to do it, there are other aspects which ALL teachers should be able to handle, regardless of their subject-matter interests and backgrounds. Essentially, the basic reading deficiencies of slow learners revolve arounc their need to simply have a better and more extensive understanding of what words mean. The formidable nature of mathematics vocabulary can be exemplified by the following definition of a replacement set taken from a 7th grade text:

> The symbols which may be substituted for a placeholder in an open sentence name members of the REPLACEMENT SET. Only, the name of a member of the replacement set may be substituted for the placeholder.

And in our current 4th grade texts, in addition to the standard vocabulary such as multiplier and divisor, we find such words as:

associative	equivalent	inscribed	parallels	identity elements
commutative	isosceles	circumscribed	diagonals	common factors
distributive	congruent	parallelogram	composites	rational numbers

Obviously, too much stress should not be placed upon such words in the lower grades. Actually, most of these words will reappear regularly in later grades. However, word understanding must precede mathematics understanding. Hence, the teacher must overtly teach "reading" as a regular part of his mathematics instruction. Not only should key words be thoroughly discussed in class, but tests should contain questions that will determine how well students have grasped the meanings of the words. One way to focus student's attention upon the importance of vocabulary is to have them keep MATH VOCABULARY NOTEBOOKS. These can be set up somewhat as follows:

WORD	MEANING	EXAMPLE
Numeral	The name of a number	five, 𝍸, 5, V
Numerator	the top numeral in a fraction	$\rightarrow \frac{3}{8}$
Denominator	the bottom numeral in a fraction	$\frac{8}{3}$ $\leftarrow$
Proper fraction	a fraction having a numerator that is smaller than the denominator	$\frac{7}{8}$; $\frac{100}{101}$

Finally, you may ask, "What are the KEY words students should know?" For many reasons, it would not be practical to prepare such a list. This is an area where the teacher must decide whether or not the vocabulary is getting in the way of understanding.

SOME WAYS TO INDIVIDUALIZE INSTRUCTION

In the April, 1966, issue of the ARITHMETIC TEACHER the following conclusions and recommendations were reported in an article entitled "A Survey of Arithmetic Intraclass Grouping Practices" by Emery Brewer:

Conclusions

It appears reasonable to draw certain conclusions from the findings of the study, an examination of the related research, and a consideration of criteria established from the written opinions of professional educators. These conclusions are:

1. The practice of grouping pupils for instruction in arithmetic within the elementary school classroom is sufficiently widespread (one out of three teachers do group) to merit further clarification of its contribution to the teaching-learning situation.

2. Intraclass grouping of pupils for arithmetic instruction is desirable as a means of aiding the individualization of such instruction.

3. A majority of teachers who group pupils for arithmetic instruction believe that such grouping aids them in individualizing instruction in arithmetic.

4. Teachers with "high" academic qualifications have a better understanding of learning principles as taught in institutions of higher learning and, therefore, see a greater need to individualize arithmetic instruction than do teachers with "low" academic qualifications.

5. Teachers with a "very high" interest in arithmetic as a subject are more likely to group pupils for arithmetic instruction than are teachers without such an interest.

6. The slow learner's self-concept is a major concern of a majority of those teachers who group pupils for arithmetic instruction.

7. The major factors that appear to contribute to a teacher's decision to use intraclass grouping for instruction in arithmetic are availability of teaching materials for use by subgroups, awareness of the existence in his classroom of a wide range of pupil differences in arithmetic learning ability, teacher's interest in arithmetic as a subject, and availability of teacher time for lesson planning for subgroups.

8. Teachers who group pupils for arithmetic instruction tend to make more provisions for pupil differences in learning arithmetic than do teachers who do not group.

9. Many teachers do not feel that they have sufficient teaching materials available to permit them to provide materials for pupil groups in arithmetic.

10. When a school employs interclass ability grouping, some teachers in the school believe that the problem of individualizing instruction in arithmetic is minimized to the extent that a need to subgroup pupils does not exist.

11. Many teachers believe that more class time is required to teach subgroups than is required to teach the class-as-a-whole.

12. Grouping for arithmetic instruction is relatively as important as grouping for reading instruction, but a majority of Ohio's public elementary school teachers do not have the professional understanding necessary to reach this conclusion.

13. Most teachers who group pupils for arithmetic instruction use only two groups.

Recommendations

1. Pupil grouping for arithmetic instruction should be encouraged as an aid in individualizing instruction.

2. The major purpose of pupil grouping, to aid in the individualization of instruction, should be fully explained to teachers.

3. Teachers should be taught that it is important to teach arithmetic to each pupil at his level of understanding and, thus, help him progress at his maximum rate, whether or not a wide range of pupil arithmetic abilities exists in a particular classroom.

4. Teachers should be apprised of the fact that it is desirable to individualize instruction in arithmetic, but that this requires a considerable amount of teacher time and should be carried out only to the point where the total program will not suffer.

5. New teachers should be encouraged to use only two groups until they become acquainted with the problems of teaching in general and the problems of pupil grouping in particular.

6. The increased demand of pupil grouping on teacher time should be explored fully and explained to teachers.

7. Since many teachers indicate that they believe more class time is required for pupil groups than for the class-as-a-whole procedure, this idea should be explored and clarified.

8. Specific attention should be given to the problem of selecting arithmetic teaching materials for pupil subgroups.

One of the major obstacles to individualizing instruction is the need to find specially prepared materials to help do the job. Within the past few years, a good deal of effort has been expended in this direction by publishers. The tendency seems to be to produce self-contained units for independent study. Houghton Mifflin has published several sets for elementary and secondary school use. In addition there are available programmed materials for almost every level of instruction. Since there are so many titles, a list of publishers has been included in the Appendix. Write them to find out what is available.

ON USING STATE-ADOPTED TEXTS

A teacher should not let himself get bound between the covers of any one book. Nothing could be moı deadening than to march through a text unrelentlessly page by page having as a primary objective "finishing the book". In all probability, the students will be finished long before the text is.

The truth is that no one set of authors should be considered to be the final word in content or approach. For example, one of our 3rd grade state-adopted texts spends a considerable amount of time developing and using the subtraction algorithm by expanded notation. Many pages are devoted to this approach:

185	means	17 tens	15 ones		
-97	means	9 tens	7 ones		
$\rightarrow$		8 tens	8 ones	or	88

The other 3rd grade state-adopted text devotes ONE PAGE to the subtraction algorithm using the standard approach:

STEP 1	STEP 2	STEP 3	STEP 4
64 -26	5 $\cancel{6}{}^{1}4$ -2 6	5 $\cancel{6}{}^{1}4$ -2 6 8	5 $\cancel{6}{}^{1}4$ -2 6 3 8
4 - 6 = !!	64 = 50 + 14	14 - 6 = 8	50 - 20 = 30

If we assume one approach to be THE approach, then the other must be inappropriate. More probably, both have some merit and some deficiencies. In the final analysis, it is the dialogue between the teach‹ and his students that should determine how well a concept or an algorithm is getting across, and how much time should be devoted to the effort. Hopefully, this guide will help in this respect.

Much is said about the "discovery approach" and most books claim to use it. However, there are several approaches which purport to be "discovery" in nature. How they differ can be seen in this excerpt from THE FLORIDA EDUCATIONAL RESEARCH AND DEVELOPMENT COUNCIL (FERDC) bulletin dated May, 1965:

The role of the teacher or instructor may be seen as the decisive force in both discovery and non-discovery learning situations. In discovery learning, although the generalization, principle or concept to be learned is withheld from the learner by the instructor, the instructor may, in varying degrees, facilitate the learning procedure. These levels of intervention by the teacher have been defined on three levels.

1. Autonomous Discovery: This is a discovery technique involving little, if any, direction from the instructor, other than possibly some basic instructions in the beginning. Neithe the learner nor his environment are manipulated once the treatment is begun. He is literally on his own.

2. Guided Discovery: This discovery technique allows the instructor to manipulate the environment but not the learner. Consequently, by being able to determine the type and sequence of data made available to the learner during the learning session, the instructor gains considerable control with respect to either facilitating or retarding the learning process.

3. Directed Discovery: Characteristic of this technique is the manipulation of the environment and student up to, but not including, giving him the so-called generalizati or concept. For example, leading questions are quite often asked by the instructor for the purpose of directing the learner's thought processes into more meaningful channel Since this is the most frequently employed discovery technique, a list of the distinguishi attributes is given below. You may notice that all but two could be used in defining discovery methods in general.

   Tends to be pupil-centered

   Acquires student participation

   Imposes responsibility on both student and teacher for continuity of development

   Employs teacher as resource agent, provocateur, encourager, and challenger.

One example of how the 3rd method can be employed is to develop the concept of how points on a line can be associated with numbers (the number line). The concept can then be further developed by plotting points on a line and a plane and showing how the concept is related to the locations of places on a map or a globe.*

*For a more detailed treatment see Francis T. Sganga, "A Bee on a Point, a Line, and a Plane", ARITHMETIC TEACHER, Nov. 1966

APPENDIX 1

TIPS ON TEACHING MATHEMATICS

1. In preparing for a class presentation on a topic, consult several texts. Some authors present text material more clearly than others. Even older texts are useful in this regard.

2. Talk to experienced teachers frankly about your concern. You can profit from their time-tested approaches to presentations.

3. Collect and file articles, pictures, puzzles, games and projects.

4. Lay out your plans for the year on a master sheet like an architect does prior to construction of a building. It will help you pace yourself.

5. Subscribe to the ARITHMETIC TEACHER (Elementary and Junior High) and the MATHEMATICS TEACHER (Secondary) by joining the National Council of Teachers of Mathematics (NCTM). Address: 1201 Sixteenth Street, N.W., Washington, D.C. 20036. Also institutions, libraries or individuals may join the Florida Council of Teachers of Mathematics by contacting the DISTRICT, Director.

6. Keep in mind that it is not the purpose of the mathematics teacher to make mathematicians of all students.

7. Lend as much variety to your teaching as possible. Sameness produces boredom.

8. Remember that students forget quickly. Reviews are needed even after a Christmas holiday.

9. Most students are very practical. Hence, practical application should be shown wherever applicable

10. Try to move from a known concept to a new concept showing the relatedness of the two.

11. Give clearly worded assignments. Place them in a special place on the chalkboard.

12. Talk as little as possible. Unless you are a good comedian, monologues are deadly. Discussions, demonstrations and interrogations are more effective. Where possible, have students do the demonstrating.

13. Procure as many good teaching aids as you can. An overhead projector is highly valued by experienced teachers. Prepared acetates can be re-used many times and students show greater interest in projections than chalkboard work.

14. Do not erase chalkboard work too quickly.

15. Be careful that students don't use you in place of reading the text.

16. Allow class time for supervised work to enable students to ask questions. Stroll down the aisles asking questions and checking work.

17. Don't demand that students do homework problems which have not been sufficiently dealt with in class.

18. Set high standards of neatness.

19. Administer frequent short quizzes.

20. Teach-test-reteach-retest.

APPENDIX 2

FINDING THE LCD FOR ANY SET OF DENOMINATORS

Sometimes in problems such as $\frac{2}{3}+\frac{1}{4}+\frac{2}{5}+\frac{1}{6}$, it is somewhat difficult to find the LCD. A fool-proof method follows: (Using the fractions just above.)

1. Line up the denominators this way:

$$\begin{array}{r|l} & 3,\ 4,\ 5,\ 6 \\ \hline \end{array}$$

2. Think of the smallest number (except 1) that can divide at least 2 of the denominators evenly. Such a number is 2. Then:

$$\begin{array}{r|l} 2 & 3,\ 4,\ 5,\ 6 \\ \hline \end{array}$$

3. Divide those denominators which are divisible by 2, placing the answers directly under the numbers being divided.

$$\begin{array}{r|cccc} 2 & 3, & 4, & 5, & 6 \\ \hline & & 2 & & 3 \end{array}$$

4. "Bring down" any numbers not divisible by 2 (such as 3 and 5 here.)

$$\begin{array}{r|l} 2 & 3,\ 4,\ 5,\ 6 \\ \hline & 3,\ 2,\ 5,\ 3 \end{array}$$

5. Enclose the "dividend" (3, 2, 5, 3) with a division symbol and again find the smallest number that will divide at least 2 of the numbers. Such a number is 3. Dividing by this number we get: (Again, "bring down" those numbers not divisible by 3.)

$$\begin{array}{r|l} 2 & 3,\ 4,\ 5,\ 6 \\ \hline 3 & 3,\ 2,\ 5,\ 3 \\ \hline & 1,\ 2,\ 5,\ 1 \end{array}$$

6. Since the "remainder" does not contain at least 2 numbers which are divisible by the same number (except 1), no further divisions are useful. The final step is to multiply all of the numbers "outside" the division symbols. The least common denominator (LCD) is 60:

$$2 \times 3 \times 1 \times 2 \times 5 \times 1 = 60 \text{ LCD}$$

FOR PRACTICE

a) $\frac{2}{3}+\frac{3}{5}+\frac{2}{9}$

b) $\frac{3}{4}+\frac{4}{7}+\frac{3}{8}$

c) $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}$

d) $\frac{1}{9}+\frac{2}{3}+\frac{7}{12}+\frac{5}{6}$

e) $\frac{1}{2}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}$

f) $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}+\frac{2}{7}$

ANSWERS

1) 1 22/45 2) 1 39/56 3) 533/840 4) 2 7/36 5) 2 53/60 6) 2 11/210

APPENDIX 3

HOW TO CHANGE BASE TEN NUMBERS TO ANY OTHER BASE

PROBLEM: Change 147_{ten} to base five.

SOLUTION:

1. Set up the problem as shown on the right. (The r stands for "remainder.")

5	147	r

2. Dividing 147 by 5 gives 29 r 2. Place these numerals as shown on the right:

5	147	r
	29	2

3. Dividing 29 by 5 gives 5 r 4. Place the numerals as shown:

5	147	r
5	29	2
	5	4

4. Dividing 5 by 5 gives 1 r 0.

5	147	r
5	29	2
5	5	4
	1	0

5. Dividing 1 by 5 gives 0 r 1. This completes the problem. Read from remainder 1 upward, we get 1042_{five}, answer.

5	147	r
5	29	2
5	5	4
5	1	0
	0	1

CHECK: $1\ 0\ 4\ 2_{five}$

$$2 \times 5^0 = 2 \times 1 = 2$$
$$4 \times 5^1 = 4 \times 5 = 20$$
$$0 \times 5^2 = 0 \times 25 = 0$$
$$1 \times 5^3 = 1 \times 125 = 125$$
$$147_{ten}$$

CHANGE THE FOLLOWING AS INDICATED:

1) 3420_{ten} to base five
2) 316_{ten} to base three
3) 97_{ten} to base two
4) 234_{ten} to base four
5) 963_{ten} to base six
6) 6241_{ten} to base eight
7) 453_{ten} to base seven
8) 5462_{ten} to base eight
9) 250 to base three

ANSWERS:

1) 102140_{five}
2) 102201_{ten}
3) 1100001_{two}
4) 3222_{four}
5) 4243_{six}
6) 14141_{eight}
7) 1215_{seven}
8) 12526_{eight}
9) 100021_{three}

APPENDIX 4

CHANGING ANY NUMBER BASE TO BASE TEN

PROBLEM: Change 263_{seven} to base ten.

SOLUTION: $2\ 6\ 3_{seven}$

$$3 \times 7^0 = 3 \times 1 = 3$$
$$6 \times 7^1 = 6 \times 7 = 42$$
$$2 \times 7^2 = 2 \times 49 = 98$$
$$\overline{143_{ten}}$$

PROBLEM: Change 11010_{two} to base ten.

SOLUTION: $1\ 1\ 0\ 1\ 0_{two}$

$$0 \times 2^0 = 0 \times 1 = 0$$
$$1 \times 2^1 = 1 \times 2 = 2$$
$$0 \times 2^2 = 0 \times 4 = 0$$
$$1 \times 2^3 = 1 \times 8 = 8$$
$$1 \times 2^4 = 1 \times 16 = 16$$
$$\overline{26_{ten}}$$

PROBLEM: Change 20132_{four} to base ten.

SOLUTION: $2\ 0\ 1\ 3\ 2_{four}$

$$2 \times 4^0 = 2 \times 1 = 2$$
$$3 \times 4^1 = 3 \times 4 = 12$$
$$1 \times 4^2 = 1 \times 16 = 16$$
$$0 \times 4^3 = 0 \times 64 = 0$$
$$2 \times 4^4 = 2 \times 256 = 512$$
$$\overline{542_{ten}}$$

FOR PRACTICE

CHANGE THE FOLLOWING AS INDICATED:

1) 102040_{five} 2) 102201_{three} 3) 1100001_{two} 4) 3222_{four} 5) 4243_{six}

6) 14141_{eight} 7) 1215_{seven} 8) 12526_{eight} 9) 100021_{three}

ANSWERS:

1) 3420_{ten} 2) 316_{ten} 3) 97_{ten} 4) 234_{ten} 5) 963_{ten}

6) 6241_{ten} 7) 453_{ten} 8) 5462_{ten} 9) 250_{ten}

APPENDIX 5

THE "TEACHER IS A GENIUS" TRICK

1. Have a student call out a four digit number. Write this on the board. (Example: 2,316)

2. Ask for another four digit number. (Example: 4,537) Place it under the above number as you would in addition.

3. Now you add a four digit number so that the second number and yours adds up to 9,999. (4,537 + 5,462)

4. Ask for another four digit number. (Example: 8,374) Place it under your number.

5. You write a fifth four digit number so that the sum of the fourth and fifth numbers is 9,999. (Example: 8,374 +1,625)

6. After a brief glance at the array of numbers, you then announce the total sum to be 22,314.

EXPLANATION: There are five numbers (addends). The second and third numbers and the fourth and fifth numbers each add up to 9,999. Borrowing 2 from the units column of the first number, we add this to the sets of numbers as follows:

2,316	=	2,316	-2	=	2,314
4,537 5,462	=	9,999	+1	=	10,000
8,374 1,625	=	9,999	+1	=	10,000
					22,314

In short, glance at the first number, subtract 2 from the units place, then add 20,000. If no one gras what you are doing at first, go through the process several times to give students a chance to discover how the trick works. This exercise can be used as a spring-board to a discussion of short-cuts in comp tations involving addends such as 9 ; 99 ; 999 ; 9,999 ; and so on.

80

APPENDIX 6

MODERN MATH FLASH CARDS

PROCEDURES:

1. Cut 5 inch x 8 inch file cards in half lengthwise.

2. Use felt pens of various colors.

3. Use large numerals, letters and symbols as follows:

$$\boxed{3 + N = 7}$$

4. Place problems and solution on the back of each card:

$$\begin{aligned} 3 + N &= 7 \\ N &= 4 \end{aligned}$$

5. Number each card in order.

6. Try not to give directions. Allow each student to think out the problem for himself.

7. Ask students to say: "N = 4" or "R = 8" and so on.

8. Occasionally, set up contests similar to spelling bees.

9. Many other number sentences may be developed using the symbols $>$ (greater than), $<$ (less than), and $\neq$ (does not equal).

10. The cards may be used as the basis for quick quizzes.

MODERN MATH FLASH CARDS (1)

Find the NUMBERS represented by the LETTERS: (2)

$3 + 4 = R$ (3)

$9 + F = 15$ (4)

$N + 8 = 14$ (5)

$14 = B + 9$ (6)

$12 = 4 + D$ (7)

$3 + 5 + 7 = E$ (8)

$3 + N + 4 = 12$ (9)

$R + 8 + 0 = 11$ (10)

$12 = 3 + 7 + N$ (11)

$21 = T + 8 + 7$ (12)

$7 - A = 3$ (13)

$B - 4 = 6$ (14)

$14 - D = 6$ (15)

$G - 7 = 8$ (16)

$7 = S - 5$ (17)

$12 = 12 - C$ (18)

$M - 0 = 3 + 9$ (19)

$17 - S = 3 + 6 + 0$ (20)

$D - 6 = 3 + 7$ (21)

Do the part in PARENTHESES first (22)

$(2 + 3) + 4 = D$ (23)

$5 + (6 + 0) = E$ (24)

$F + (4 + 0) = 7$ (25)

$(3 + 5) + (6 + 4) = G$ (26)

$(4 + 2) + Q = 14$ (27)

$K = (8 + 4) - (3 + 4)$ (28)

$(6+7) - J = (4+3)$ (29)

$(3+2) + (8-3) = L - 5$ (30)

What does the $\cdot$ mean in the following? (31)

$3 \cdot 4 = 12$ (32)

$A \cdot 6 = 18$ (33)

$27 = C \cdot 3$ (34)

$9 \cdot D = 54$ (35)

$56 = 8 \cdot F$ (36)

$8 \cdot G = 72$ (37)

$48 = 6 \cdot H$ (38)

$7 \cdot K = 63$ (39)

$3 \cdot 4 \cdot 5 = N$ (40)

$2 \cdot 3 \cdot R = 24$ (41)

$3 \cdot T \cdot 4 = 0$ (42)

$N \cdot 4 \cdot 2 = 48$ (43)

$(4 \cdot 5) + (3 \cdot 2) = A$ (44)

$(6 \cdot 7) + B = 50$ (45)

$C - (7 \cdot 7) = 10$ (46)

$D - (4 \cdot 7) = 0$ (47)

$6 \cdot E = (3 \cdot 2) \quad 6$ (48)

$A \div 7 = 5$ (49)

$24 \div B = 4$ (50)

$E \div 6 = 8$ (51)

$28 \div F = 7$ (52)

$B\overline{)\underline{35}}$
$\quad 7$ (53)

$6\overline{)\underline{N}}$
$\quad 9$ (54)

$\quad 9$
$7\overline{)A}$ (55)

$\quad 4$
$D\overline{)36}$ (56)

$9\overline{)E}$ with quotient 9 (57)	Q • R = 11 (71)
$F\overline{)48}$ with quotient 6 (58)	V • W = 13 (72)
Now find all of the PAIRS of numbers (59)	A PRIME number is ONLY divisible by ITSELF and 1. (73)
X + Y = 5 (60)	Pick out the PRIME numbers. (74)
7 = A + B (61)	2, 3, 4, 8, 11, 17 (75)
M + N = 10 (62)	5, 7, 9, 13, 15 (76)
R • S = 4 (63)	6, 12, 19, 21, 31 (77)
8 = C • D (64)	23, 25, 29, 33, 37 (78)
F • G = 12 (65)	Find the NUMBERS represented by the LETTERS: (79)
24 = K • L (66)	$\frac{10}{2} = A$ (80)
H • L = 3 (67)	$\frac{15}{N} = 3$ (81)
M • N = 5 (68)	$7 = \frac{B}{3}$ (82)
L • P = 7 (69)	$\frac{36}{9} = C$ (83)
S • T = 9 (70)	$9 = \frac{D}{7}$ (84)

$\frac{72}{E} = 9$	(85)	$\frac{D}{35} = \frac{4}{5}$	(98)
$8 = \frac{F}{6}$	(86)	$\frac{27}{E} = \frac{3}{7}$	(99)
$\frac{(8+7)}{3} = G$	(87)	$\frac{8}{1} = \frac{C}{8}$	(100)
$\frac{(3 \cdot 6)}{(48 \div 8)} = H$	(88)	$\frac{(3 \cdot 2)}{7} = \frac{30}{F}$	(101)
$J = \frac{(63 \div 7)}{(3 \cdot 3)}$	(89)	$\frac{2\ 1/2}{6} = \frac{N}{12}$	(102)
$30 \div (6-1) = N$	(90)	$\frac{B}{4} = \frac{1\ 1/2}{2}$	(103)
$C \div (11-1) = 5$	(91)	$\frac{22}{7} = \frac{C}{3\ 1/2}$	(104)
$25 \cdot M = (50 \div 2)$	(92)	$\frac{6}{3} = \frac{3}{E}$	(105)
$(4 \cdot 6) \div W = 8$	(93)	$\frac{7}{D} = \frac{14}{5}$	(106)
$P \div (8+2) = 3$	(94)	$\frac{100}{1} = \frac{A}{100}$	(107)
$\frac{1}{2} = \frac{N}{6}$	(95)	$N - 2 = (1/2 + 1/2)$	(108)
$\frac{2}{3} = \frac{4}{A}$	(96)	$\frac{10 + 2\ 1/2 + 2\ 1/2}{N} = 3$	(109)
$\frac{B}{8} = \frac{3}{4}$	(97)	THE END	(110)

MATHEMATICS PROJECT IDEAS

I. Models

Construct a model which illustrates a theorem, shows the meaning of a formula, explains a mathematical principle, or represents a measuring device. After you have constructed your model, try to make some use of it. Perhaps you can demonstrate it to the class, make a series of readings with it, or use it to collect data. In the list below you will find some suggestions for these models. As you read these suggestions, you may get an idea for another kind of model which is not mentioned.

1. Wooden model to show simple and compound interest.

2. Homemade slide rule. Include in the write-up a method of determining values.

3. Homemade sextant. Use this instrument to "shoot the sun" and determine the latitude of your town.

4. Cross-staff. Use a ruler and calling card. Problems in proportion can be solved with this instrument.

5. Altimeter. Use a large piece of cardboard, a string, and plumb bob. Measure inaccessible distances.

6. Models to show the meaning of formulas. For example, to show the area of a circle, cover a wooden box 2" deep with glass. A raised partition divides the bottom of the box through the center. In one side is a depressed circle, in the other a depressed area composed of 3 - 1/7 squares with length of the radius as the side of each square. The surface of the circle is completely covered with shot which can be poured over to the other side and shaken down so that it exactly covers the surface of 3 - 1/7 squares.

7. Wheel device for showing the line values of the trigonometric functions.

8. Model house. Use scale drawings and geometric principles.

9. Telemeter, a device for measuring distances. (Reference: Mathematics Teacher, November, 1955, pp. 473-474.)

10. Models to illustrate theorems. For example, make a model visualizing the Phthagorean Theorem. (Reference: Mathematics Teacher, April, 1955, pp. 246-247; November, 1955, pp. 475-476.)

11. Demonstration board which illustrates the largest area that can be enclosed with a fixed perimeter.

12. Device which illustrates the use of motion in forming geometric figures. (Reference: Plane Geometry, by Keniston and Tully, p. 221.)

13. Homemade transit. Find the height of the school flagpole.

14. Elevator to be used in the study of addition of signed numbers.

15. Geometric solids of lucite.

16. Movable wooden model with three concentric circles showing fractions, decimal equivalents, and per cents.

17. Models which demonstrate how areas and volumes of various figures are determined.

18. Bank. Show its parts and departments.

19. Movable models to illustrate locus.

20. Balance with numbered weights to illustrate elements of algebra and rules of equality.

21. Model showing relationship of multiplication to addition.

22. Three dimensional set of models showing uses of mathematics in industry.

23. Models showing relationship between base, percentage, and rate in per cent problems.

24. Models which illustrate the steps in problem solving.

25. Illustrate with wood: $(a + b)^2 + a^2 + 2a + b^2$

26. Pendulum. Does the equation stating the relationship of a pendulum's length to its period hold true for pendulums of any length?

27. Gear train. Compute a gear train that would be suitable for a clock on Mars, assuming a 24-hour day is desired, and a one r.p.m. motor is the power source. Make a large scale drawing of the drive.

28. Bridge. Make a model bridge of toothpicks to illustrate geometric design.

29. Wire models of figures of revolution and conic sections. Paint to point out different parts.

30. Models of solid figures indicating slant, height, altitude, and the like.

31. Inverted double cones rolling uphill. Illustrate center of gravity.

II. <u>Collections</u>

You may wish to make a collection which shows the use of mathematics in everyday life, illustrates a formula or theorem, or explains a mathematical principle. It is important to keep one of these purposes in mind when deciding upon a collection.

1. Photographs or drawings showing the uses of lines and angles in everyday life.

2. Measuring instruments used in industry.

3. Geometric designs placed on wood and cut out.

4. Postage stamps which have reference to mathematics. (Reference: <u>Mathematics Teacher</u>, November, 1955, p. 477.)

5. Various measuring instruments found in the home.

6. Pictures of famous mathematicians with frames in the shape of triangles or other geometric figures.

7. Set of colored slides showing relationship of art and mathematics in a church's stained glass window.

8. A collection showing the development of measurement through the ages.

9. A collection to show:

   Mathematics and design
   Mathematics and bridges
   Mathematics and maps
   Mathematics and defense
   Mathematics and churches
   Mathematics in the army and navy

III. Research

There are many interesting topics in the field of mathematics for research and study. Perhaps you may wish to select one of these topics and begin your study of it. Try to present your findings in an interesting and illustrated manner.

1. Mathematics in music. Include a study of the development of a particular instrument.

2. Geometry in range finding and ballistics.

3. Development of our number system.

4. Symbolism in mathematics.

5. Heroes in the field of mathematics.

6. A history of mechanical devices for drawing curves. Begin with Nicomedes (270 B.C.).

7. Development of instruments of indirect measurement.

APPENDIX 8

ADDITIONAL PROJECTS IN MATHEMATICS

Infinity:
- Critical analysis of infinity
- Galileo's ideas concerning infinity
- The notion of infinity in the work of the Lation poet Lucretius

Integrals, physical application of

Kaleidoscope

Knots

Leibaiz, biography of

Level:
- Egyptian level
- Use of the level

Limit -- models or diagrams to illustrate the concept of limit

Linkages
- Linkages in the third dimension

Locus:
- Determination of the locus of a point on a circle rolling on another circle which itself rolls on a straight line

Locus problems

Logarithms:
- History of logarithms
- Various systems of logarithms

Magic Knight's Tour (chess)

Magic squares

Mathematical cartoons -- collected from newspapers and magazines or original

Mathematical terms, analysis of as related to classical physics

Mathematical thought as related to the culture of a civilization

Mathematics:
- How mathematics is used in (any selected occupation)
- Influence of mathematics on the social sciences
- Mathematics as an abstract creation
- Mathematics as an art
- Mathematics in art
- Mathematics and color
- Mathematics and depreciation
- Mathematics of the Egytians, Babylonians, and Phoenicians
- Mathematics in gears
- Mathematics of the Hindus and the Arabs
- Mathematics in music
- Mathematics in nature
- Mathematics in the news; collection of newspaper clippings about mathematics
- Mathematics in photography
- Place of mathematics in the formation of physical theories
- Renaissance mathematics
- Various views on nature of mathematics
- Maxima and minima

Measurement:
- Importance of the straight line in measurement
- Indirect measurement (problem and models)
- Measuring distances on earth

Mendel's theory, mathematics of

Mixture problems, demonstration of

Mathematical ideas, source of

Mathematical principles of particle acceleration

Mathematical series and vanishing triangles

Mathematical surfaces, properties of

Mathematical system -- the resemblance of the American Declaration of Independence to a mathematical system

Napier:
- Biography of Napier
- Napier's Bones

Newton, biography of

Nomographs -- solving simultaneous equations

Normal distribution:
- Graph on normal distribution of height (including collection of data)
- Graph on normal distribution of lung capacity (including collection of data)
- Graph on normal distribution of weight (including collection of data)
- Normal distribution as illustrated by sea shells

Numbers:
- Approximate numbers
- Duodecimal system
- Finite number system
- History of Arabic numbers
- History and practical applications of number systems
- Names of numbers in different languages
- Number e
- Number theory
- Patterns of numbers
- Prime numbers
- Problems and illustrations of a number raised to a power, such as grains of wheat on a checkerboard or folding a newspaper

Models:

To show geometric interpretation of $(a + b)^2$
To show geometric interpretation of $(a - b)^2$
To show geometric interpretation of $(a^2 - b^2)$
To show how "64-65" in the cut-out checkerboard problem
To show relationship of volume of cylinder and cone

Mosaics by reflection

Pi:
- Determining the value of pi by the measurement of a circle
- Probability determination of pi

Pictures of persons in different occupations and how they use mathematics

Plane curve theory

Plane table

Planimeter

Polygons:
- Investigation of the properties of the diagonals of polygons
- Regular polygons
- Regular 17-sided polygons

Probability board

Problems of antiquity: squaring a circle; trisection of an angle; duplication of a cube

Projectiles

Proof -- charts showing how a proof is based on definitions, axioms, and postulates

Pythagoras:
- Biography of Pythagoras
- How the Egyptians used the theorem of Pythagoras
- Model to show Pythagorean relationship
- Pappus' extension of the Pythagorean theorem

Optical illusions

Osculating plane -- studying the twist and turn of a third-dimensional curve by use of the osculating plane

Pantograph

Paper folding

Parabolic reflector

Pascal's theorem

Pendulum
- Pendulum designs

Saccheri, life and work of

Satellites

Scale drawings (as applicable to photography)

Sequences:
- Sequences and the number
- Summation of sequences

Sextant

Shell forms, mathematics of

Slide rule:
- Construction of slide rules
- History and operation of the slide rule
- Quadratic slide rule
- Slide rule and logarithms
- Slide rule for positive and negative numbers

Snowflake

Soap bubbles -- the question of curvature

Spiral

Statistics -- how to lie with statistics

Surfaces:
- Minimal surfaces
- Ruled surfaces

Peaucellier's cell

Quadratics, geometric construction of

Rates of change

Ratio -- illustrations and applications of the concept of ratio

Reasoning fallacies in advertising

Reflections, repeated

Reflex arc

Relativity

Tessersat

Suspension bridge and the catenary curve

Symbolic logic

Symbols for numbers

Symmetry (meaning, types, illustrations, models)

Surveying transit

Tangrams -- geometric dissections

Tetrahedron tower

Thales, biography of

Theorem of residuea -- use of in evaluation of certain definite integrals

Theorems -- models to illustrate theorems

Time curves

Topology -- Moebius strips and other simple topological problems

Trapezoid model to show area of trapezoid

Triangles:
- Geometric construction of triangles
- Model to show area of triangle
- Models or diagrams to show the four centers of a triangle and methods of determining them
- Practical application of the laws of triangles

Variation -- practical use of variation in price charts rules of thumb in mechanical trade, gardening, etc

Volume of an irregular solid, determining of

Trigonometric functions:
- Trigonometric functions of imaginary and complex angles
- Trigonometric functions -- the unit circle

Trigonometry:
- History of trigonometry
- Navigation problem with model
- Use of trigonometry in navigation

Vector analysis

Zeno and his school of philosophy

abscissa

the first number in an ordered pair, sometimes called the x coordinat
$(\underline{3}, 5)$ $(\underline{x}, y)$

additive inverse

for every whole number, such as 3, there is another, such as -3, wh
when added give the sum 0. $3 + (-3) = 0$

addend

the name given to numbers being added. In $4 + 3 = 7$, the 4 and
are ADDENDS.

algebra

generalized arithmetic. (If $A + B = C$, then $C - B = A$, etc.)

algorithm (algorism)

a computational process, such as multiplication, division, or finding
square root.

analytical geometry

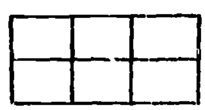
mapping numbers as points on a graph, converting equations into geo
shapes, and converting shapes into equations.

arithmetic

the science of numbers

array

an orderly arrangement of elements by rows and columns:



2 by 3 array

* * * *
* * * *
* * * *

3 by 4 array

associative (law)

addition: In adding 3 numbers, grouping does not affect the sum:
$3 + (4 + 2) = (3 + 4) + 2$

multiplication: In multiplying 3 numbers, grouping does not affect the
product: $3(4 \times 5) = (3 \times 4)5$

assumptions

statements accepted without proof

average

see MEAN

axes (singular: AXIS)

two intersecting perpendicular lines which serve as reference lines for
graphs

axiom

a basic assumption or postulate. Example: If equals are added to equ
the sums are equal. Two straight lines can cross only once.

base

the number upon which a numerative system is constructed:

10 digits	=	decimal system	=	base ten	=	10^3	10^2	10^1	10^0
5 digits	=	quinary system	=	base five	=	5^3	5^2	5^1	5^0
2 digits	=	binary sytem	=	base two	=	2^3	2^2	2^1	2^0

binary

1. Numeration system using only two digits, 1 and 0.
2. Operation performed on 2 numbers. Addition is a BINARY OPERATION, since only 2 numbers can be added at a time.

calculus

a system for analyzing change and motion in terms of points or numbers strung together in continuous sequences. It helps explain such things as the motions of planets and space ships.

closure

Example: whole numbers are CLOSED under the operations of addition and multiplication (the sum or product gives another whole number), but NOT CLOSED under subtraction and division ($5 - 7$ and $5 \div 7$ do not give whole numbers).

commutative (law)

addition: In adding 2 numbers, the order of addition does not affect the sum: $3 + 4 = 7 = 4 + 3$

multiplication: In multiplying 2 numbers, the order does not affect the product: $3 \times 4 = 12 = 4 \times 3$

complex fraction

one having one or both terms as fractions or mixed numbers:

$$\frac{1/2}{3} \qquad \frac{2/3}{3/4}$$

composite number

any natural number which is not PRIME. (4, 6, 8, 9 . . .)

congruent

(means same size and shape.) In congruent triangles, the corresponding sides are equal. Two angles are congruent if they have the same measure.

constant

any number that never changes in a relationship such as $C/d = \pi$ where pi equals about 3.14. (Also see: Pi and Function)

coordinates

coordinates of a point are the ordered pairs of numbers which designate the point.

deductive reasoning

a conclusion that is "deduced" by logical reasoning from accepted statements (assumptions). This method is used in geometric PROOFS.

digit

the numerals usually used to represent numbers. In base ten, these are 0 through 9. The number of digits in any base is the same as the base number. The lowest digit is always 0. In base five, the five digits are 0, 1, 2, 3, and 4.

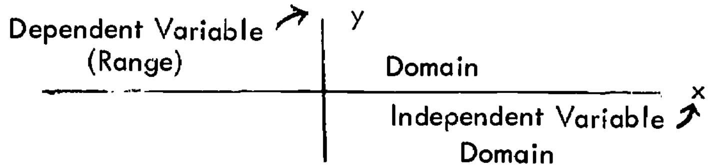
distributive property (law)

the distributive law holds for multiplication over addition: If a, b, and c are any numbers, $a(b + c) = ab + ac$ . . . and $a(b + c) = ab + ac$

divisible

one natural number is divisible by another when the remainder is 0.

domain	is the ser of numbers which can replace x, the independent variable in such equations as $y = 2x + 4$.



expanded numerals

numeral	Expansions
234	200 + 30 + 4
	$(2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$

exponent	tell how many times base is to be used as a factor
equivalent	having the same value but differing in form such as in 1/2 and 2/4.
face value	the value of a digit.
factor	numbers that are multiplied
factor tree	a method of showing the prime factors of a number

$$\begin{array}{c} 24 \\ 6 \times 4 \\ 2 \times 3 \times 2 \times 2 \end{array}$$

formula — is a statement in symbols which expresses a relation between quantities.

$\frac{C}{d} = \pi$ (math) H_2O (chemistry) $\frac{d}{t} = v$ (physics)

fraction — an indicated quotient of 2 quantities, denominator not zero. The 2 quantities are called TERMS, or numerator (top term) and denominator (bottom term).

Fractional Situations:

1. One or more equal parts of a whole.
2. One or more of the parts of a group.
3. A comparison or ratio.
4. An indicated division.

function — consider $C = \pi \times d$. For each value of the independent variable d, there is only 1 value of the dependent variable C. Here, C is a "function of" (depends upon) d. When d is 3 1/2, C is 11; when d is 7, C is 22 and so on. (Also see: CONSTANT and Pi.)

Also, the distance traveled by a car is a FUNCTION of two variables: time and speed. $(D = r \times t)$

geometry	the science of shapes and spatial relationships.
graph	a way of picturing data so that rough comparisons can be easily made.
greatest common factor	Also called GREATEST COMMON DIVISOR. It is the largest factor common to two or more numbers. The GREATEST COMMON FACTOR of 18 and 24 is 6.
identity elements zero and one	<u>Additive Identity Element:</u> zero zero added to any number gives the same number <u>Multiplicative Identity Element:</u> 1 The product of any number and 1 is the same number significance: $\frac{2}{3} = \frac{2}{3} \times 1 \qquad \frac{2}{3} \times \frac{3}{3} \qquad \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$
integers	the subset of rational numbers consisting of whole numbers and their additive inverses (negatives): $(\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots)$
inductive reasoning	(experimental method) The reasoning process in which a conclusion is based on one or more (but not all) cases. (EXAMPLE: The determination of π by dividing the measure of the diameters of circles by their diameters.)
inverse operation	one operation (such as subtraction) which undoes another (addition). inverse of $3 + 4 = 7$ is $7 - 4 = 3$ inverse of $3 \times 4 = 12$ is $12 \div 4 = 3$
irrational numbers	numbers which cannot be expressed as ratios between 2 integers. ($\sqrt{2}$; π)
least common denominator (LCD)	given a set of fractions, the LEAST COMMON DENOMINATOR (LCD) is the smallest natural number divisible by each of the denominators. LCD for $1/3 + 1/4 + 1/5 + 1/6$ is 60 See Appendix on finding the LCD for any set of denominators.
least common multiple (LCM)	the smallest natural number divisible by each number in a set. If the numbers are DENOMINATORS, the LCM is the same as the LCD.
literal numbers (also literal equations)	1. the letters (variables) used to represent numbers. 2. literal equations contain only letters
mixed number	an integer (whole number) plus a fraction 3 1/7.
an (average)	the sum of all the scores divided by the number of scores.

median	the middle score in a set of scores arranged in order of size.
mode	the score which occurs most frequently in a set of scores
modular arithmetic (also clock or finite arithmetic)	based upon a finite (limited) set of numbers. Examples are a clock or telephone dial.
multiplicative inverses (see reciprocal)	two numbers whose product is 1.
natural numbers	counting numbers from 1 to infinity.
notation	using symbols to represent numbers.
number sentence	see SENTENCES
numeral	the NAME given a number. Five things may be represented by the numerals: 5 five V 卌
numeration	the process of presenting numbers in order of size
open sentences	see SENTENCES
opposite numbers	see ADDITIVE INVERSE
ordered pair	a pair of numbers whose ORDER is important. EXAMPLES: 1. (3, 4) point on a plane 2. (3, 4) ratio 3:4 or 3/4
ordinate	the second number of an ordered pair (3, <u>4</u>) sometimes called the y coordinate.
origin	the point labeled 0 on a number line or (0, 0) on a 2-dimensional graph where the axes intersect. The origin is a POINT OF REFERENCE.
pi	the number derived from dividing the circumference of a circle by its diameter. π = 22/7 or 3.14159 . . . (Also see: CONSTANT and FUNCTION)
place holder	a symbol that holds a place for a numeral EXAMPLES: N and $\square$ in: 4 + N = 7 $\square$ - 3 = 4
place value	(Also, NUMERICAL VALUE.) The value of a digit due to its relative position in the numeral.

plane	usually an undefined term but describea as a two-dimensional geometric surface.
postulates	see AXIOM
prime factor	a prime that divides a number
prime number	a number greater than 1 that is divisible only by itself and 1.
principles	mathematical laws. See ASSOCIATIVE, COMMUTATIVE and DISTRIBUTIVE.
property	an attribute or characteristic of a class. For example, it is the property of natural numbers that the sum of any two of them is again a natural number; this is generally referred to as the Closure Property of Addition. The structural properties of arithmetic are the Associative Property of Addition, the Associative Property of Multiplication, the Commutative Property of Addition, the Commutative Property of Multiplication, the Distributive Property of Multiplication in respect to Addition, the Identity Elements (0 for addition and 1 for multiplication), the Inverses, and the Closure Property. Other properties are EVEN, ODD and divisibility. All 45° right triangles have the same PROPERTIES regardless of size.
proportion	a proportion is a statement that 2 ratios are equal.
quotient	result of division.
range	refers to the set of numbers which can replace y, the dependent variables in equations such as $y = 2x + 4$.
rate	1) a ratio of one quantity to a unit of a related quantity. $\frac{\text{30 miles}}{\text{1 hour}}$ $\frac{30}{\text{1 ft.}}$ $\frac{\text{25 students}}{\text{1 class}}$ $\frac{5\%}{\text{1 year}}$ The horizontal line is read "per" as in 30 miles per hour.
ratio	the comparison of 2 numbers or the quotient of 2 numbers.
rational	any number a/b where a and b are integers and $b \neq 0$. Intergers are also RATIONAL NUMBERS since they can be expressed in the form N/1 where N represents any integer.
reciprocal	If the product of a number is 1, the numbers are RECIPROCALS of each other. EXAMPLES: 2 and 1/2 3/4 and 4/3
renaming	$6 \times 9 \rightarrow (2 + 4) \times 9 \rightarrow (2 \times 9) + (4 \times 9)$

replacement set

a set whose elements serve as the replacement set for the variable. T set is also the DOMAIN of the variable.

statement

see SENTENCES

sentences

	STATEMENTS		
open sentence	true sentence	false sentence	
3 + N = 7	3 + 4 = 7	3 + 4 = 5	Equ.'ions
x > 4	7 > 4	7 < 4	Inequalities

sets

a SET is a collection of objects which are called ELEMENTS or MEMB

equal sets

contain precisely the same elements.

equivalent sets

the elements can be matched one-to-one with none left over; the num of elements in each set is the same.

intersection

the intersection of 2 sets consists of those elements which are common both sets. The symbol for intersection is $\cap$

union

the union of 2 sets A and B is a set such that all of the elements of A all of the elements of B are in the new set, but elements common to b sets are not repeated.

similar

same shape but not necessarily the same size.

theorem

theorems are deduced from the assumptions, postulates, and definition upon which there has been agreement.

trigonometry

deals with the relations existing between the sides and angles of trian

undefined terms

key words which are understood by all persons in a particular discussi and yet are not defined.

point line (?) congruence (?)

variable

a place holder or unknown in an equation or an open sentence. In $C = \pi \times d$, the value of the variable C depends upon the value of th variable d. Therefore, C is the DEPENDENT variable, while d is the INDEPENDENT variable.

vector

a line segment that represents both DIRECTION and SIZE.